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# A Schumpeterian Model of Duopolistic Competition

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**ABSTRACT**

**Aim**. This paper models the dynamics of technological change through the competitive interaction of two firms. The duopolists strive to outperform each other by exploiting the two fundamental Schumpeterian forces of economic development: innovation and imitation. **Method**. By extending over a number of periods a technological "limit-pricing model" (whereby the "learning-by-doing effect" is the source of the barrier to entry) and assuming that the two firms compete following one another in the role of innovator and imitator. As **result**, it is possible to trace out the paths followed by the market shares of both producers and to derive endogenously a unit cost curve characterizing the industry in the long run. **Conclusion**. A further merit of the model presented herein is its representation of a "micro-macro transition phase" — viz., the passage from individual practice to industrial standard — through a simplified but, nonetheless, *realistic* depiction of behavioural routines.

***Keywords:*** Industrial organization; firm behaviour; innovation; imitation; technology; technological change; Schumpeter; dynamics; learning economies; paradigm shift; industrial competition; duopoly

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ОРИГИНАЛЬНАЯ СТАТЬЯ

# Шумпетеровская модель дуополистической конкуренции

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**АННОТАЦИЯ**

**Целью** статьи является презентация **метода** моделирования динамики технологических изменений посредством конкурентного взаимодействия двух фирм. Дуополисты стремятся превзойти друг друга, используя две фундаментальные шумпетеровские силы экономического развития: инновации и подражание. В **результате**, распространяя на ряд периодов технологическую «модель предельного ценообразования» (где «эффект обучения в процессе работы» является источником возникновения барьера для входа) и предполагая, что две фирмы конкурируют друг с другом в роли первопроходца и последователя, можно проследить пути, по которым следуют рыночные доли обоих производителей, и вывести эндогенно кривую удельных издержек, характеризующую отрасль в долгосрочной перспективе. Авторы делают **вывод**, что достоинством представленной модели является характеристика «фазы микро-макро» перехода, который, в отличие от перехода от индивидуальной практики к промышленному стандарту, посредством упрощенных, но, тем не менее, реалистичных рутинных действий.

***Ключевые слова:*** промышленная организация; твердое поведение; инновации; имитация; технологии; технологические изменения; Шумпетер; динамика; обучающаяся экономика; смена парадигмы; промышленная конкуренция; дуополия

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## 1. Introduction

This article is divided into four parts. The first presents the background of our problem, namely that of building a model that portrays an industry's evolution highly sensitive to the effect of technological pressure. Our main propositions, which will be derived from Schumpeter's ideas on the nature of development, are an attempt to provide further insights into the economic impact of innovation. The ground plan for this analysis — the second part of the paper — is set by looking first at one possible way of tackling this problem: a limit-pricing model by Dosi will be briefly described and examined. Two basic equations of Dosi's pricing mechanism will be retained in the construction of our own model; these two equations will be complemented by a group of assumptions that will radically alter the original strategic framework and give rise to a conceptually different perspective on the dynamics of competition. The third section is the exposition of the model. Our general scheme allows for several possible scenarios: the parameterization and the following discussion, which conclude the paper, centre on the special case for which the same two firms alternate "endlessly" in the role of innovator and imitator.

## 2. Background

In the vision of Schumpeter [1], the engine of modern capitalism is propelled by a complex and devastating force: technology. Human discoveries aiming at increasing societies' overall productivity through sophisticated machinery and the reorganization of labour along new technical guidelines behave like seismic waves that disrupt old productive routines and social conventions built on these economic foundations. The industrial terrain is thus forced to make room for new devices, which will reshape the system and alter the nature of the economic network. The trajectory traced out by this sequence of innovative shocks portrays the essential dynamics of competitive markets. Creativity gives birth to a novel "solution", and growth follows: the newly marketed technology evolves, reaches maturity and finally decays as the next revolutionary idea supersedes it. The life-cycle of every innovation is depicted by the well-known L-shaped, "Learning-by-doing" curve, which features unit costs as a decreasing function of time and output. In *time,* men learn as they produce more: they learn, modify and improve the product. The prolonged energy and effort — both manual and conceptual — devoted to the development and perfection of a pathbreaking idea are punctuated by that trial-and-error scansion which progressively lowers the cost of production. The cumulative path of industrial transformation translates into a sequence of several learning-by-doing descents, each breaching out at a higher level of technical efficiency than the previous one. Such a path is smoothened by the "polishing" effect of *imitation:* any major innovation is followed by a number of incremental steps which represent adjustments and refinements of the original concept. From the microeconomic perspective, depending on the revenue flows, the level of knowledge acquired, and the industrial sector's history, a producer can choose to innovate or imitate. By linking up in time these choices, one obtains the *industry's macro-dynamic picture,* which shows how swathes of successive innovations spawn clusters of technical refinements that give rise to an *aggregate learning curve.* We shall provide a rather coarse description of this transition from the micro-state (the set of routines adopted by agents) — namely, the R&D programs enacted within the corporate fences, the ensuing learning-by-doing vicissitudes, and the mixed attempts pursued by rivals to copy the innovative solution — to the *macro*-state, that is, the aggregate learning curve characterizing the industry, by narrowing the focus of the analysis on two rival firms seeking to dominate the market by means of technological advance.

## 3. A Limit-Price Model

The duopolistic scenario presented here draws partly on a *technological* limit-pricing model formulated by Dosi [2]. Dosi assumes that two agents compete: on one side, a Schumpeterian entrepreneur takes advantage of his monopolistic position; on the other, an imitator who attempts to enter the market, after a period T since the date of innovation with a copy of the original, innovative solution, produced at a lower cost than that of the innovator. Faced with this possibility, the innovator will have to consider a trade-off between higher *short-run* profits if he allows entry at time *T* (since he may charge a high price from 0 to *T)* versus higher *long-run* profits if he foregoes an immediate monopolistic rent temporarily, impeding entry effectively.

In other words, the former strategy implies that a monopolist desiring to make the most out of his protected invention by selling it at a high price immediately will shortsightedly hurt his position in the longer term by allowing, in fact, the imitator to make a profitable entry in the market: the cheaper copy, as soon as it is marketed, will eventually drive the innovator out of the market. It is what is meant by short-run profits: in the long-run, when an imitation of the original idea has been perfected, the ex-monopolist is eliminated. The second course of action, instead, allows the innovator to bank on the learning factor and force down, in time, his average cost of production, and thus the selling price: at time *T,* the Schumpeterian innovator would then find himself more competitive than a potential imitator. The threat of (hostile) entry is thus kept at bay by concentrated effort on technological inventiveness.

Therefore, if the monopolist adopts a limit-pricing strategy, he will charge at time 0, conscious that very likely he will lose competition at time *T,* a price which is just below the desired level of the entrant at that time, and yet one which guarantees a *general markup* — computed over the cumulated production from 0 to *T* — high enough to compensate the high costs of innovation. Evidently, the markup of the innovator, $m_i$, must be greater than the price-cost margin $m_m$ which ensures the imitator the minimum level of profitability. "If learning curves are sufficiently steep, this condition will always be met" (2, p. 123). In other words, counting on the possibility to "come down along" his learning curve, the monopolist will be able to recover the initial costs by means of the limit-price. Dosi encapsulates the "learning-by-doing effect" by assuming that the formula for determining the price level is cumulative production, β is the learning coefficient, $c_0$ is the initial cost of production level, and $m_i$ is the mark-up over cost. The above formula says that the price level decreases as the cumulative production increases: as a firm produces more, it learns more and therefore abates costs, and is finally able to charge a lower price for the good (this is the essence of the inverse relationship between $P_t$ and $X_t$). Dosi also assumes that the demand function has a constant elasticity:

$$x_t = f\left(P,t\right) = AP_t^{-\alpha},$$

where A is the market size. The production cost of the entrant is given by the following expression:

$$c_e = c_0 g\left(t\right).$$

It is, *by assumption***,** equal to that of the innovator minus a certain percentage *g(t)* based on the "watch-and-learn" effect. If the monopolist does not engage in learning, the imitator can eliminate the leader after a time *T* if $c_0 X_t^{-\beta} - c_0 g\left(T\right) > 0$. On the other hand, if he decides to exploit learning economies, he will set the limit-price equal to:

$$P^* \le m_m c_0 g\left(T\right).$$

Let $m^*$ be the limit-price mark-up defined by:

$$m^* = \frac{P^*}{c_0 X^{-\beta}}.$$

Therefore, the limit price markup, in this model, $m^*$, depends on:

$m_m$ the minimum rate of profit for the entrant,
T, the imitation time-lag,
β, the "learning-by-doing" coefficient,
g(t), the "learning-by-imitating" curve,
aA, the absolute size of the market.

## 4. The Model

Now suppose that two firms coexist in an industry in which one gets the jump on the other through an early innovation. At this point, we are varying the **scenario** sensibly by *assuming that the monopolist is somehow* **forced** to *face a* more *competitive imitator at time T.* It means that the strategic component, namely the mark-up, ceases to have any significant role in our model. The new underlying idea is that imitators always manage to enter the market, as the word implies. The pressing forces behind technological refinements — our notion of imitation — are irresistible.

By making this critical change, the model's focus which will be developed hereafter will no longer be predicated on barriers to entry but rather on the effects of "learning-by-doing" upon the dynamics of a hypothetical, duopolistic industry. In this new framework, we will not see a producer setting a price based on his expected technological performance; instead, producers will try to calibrate, to their own advantage, the level of investment necessary to carry out the two basic strategies of industry (in this highly rarefied theoretical scheme): imitation and innovation. Thus, this is not a setting in which the firm determines once and for all the course of action that will shape the environment, but rather one in which the firm, having only two options at its disposal (innovation and imitation), sets out to enact either one when conditions allow it (these will be explained in detail) in order to "stay ahead of the game." In other words, the central issue here is not the impact of a certain decision (as it is for limit-pricing models), but the cumulated industrial dynamics triggered by agents who will make predictable use of two basic productive strategies (again, innovation and imitation).

The assumptions of the model are the following:

*Assumption 1:* The strategic variable, the markup, no longer plays an important role: the dynamics of the model will be determined exclusively by the technological achievements of the competitors. These will try to dominate the market by means of greater technological knowledge. There is no way to prevent the competitor from improving the original idea. Although the present model will make use of a set of equations somewhat similar to that used by Dosi, it cannot be viewed as a variation on the theme of limit-pricing, for no weight is given to the markup.

Specifically, we do assume that firms earn a profit margin above their production costs but that these margins are roughly equal for both firms: *what makes* the *difference is the absolute cost level*. The more efficient producer, the one who controls the more sophisticated machinery, will be able to undersell the rival, and the way to look at this phenomenon is to concentrate the attention on the unit cost function of both firms: the presence of a profit margin is understood, but its relevance in the economy of the model is nil.

*Assumption* 2: The inevitable entrance of a competitive imitator (i.e., with a cheaper copy of the original idea) at time *T* will not lead to the immediate demise of the innovator: owing to factors such as market inertia, brand loyalty and long-dated contracts, to cite a few examples, we assume that the innovator, once the competition starts, is not ousted outright from the market but continues to meet the demand *in inverse proportion to his production costs*. That is, at any moment in time, there are two market shares, one for the firm *a* and one for firm *b*. These shares stand in a ratio inversely proportional to that of the firms' unit production costs (the higher the cost, the lower the share).

To grasp the essence of the model, the reader has to bear in mind that both firms compete in order to provide a certain service; they strive to satisfy a determined need of the consumers: thus, they cater to demand — which represents a *single* well-specified want — with *two* (for this particular model; there may be *n*) different, technologically competing products (e.g., transistors and semiconductors both satisfy the demand *for* computation, just as, for listeners, vinyl records and compact discs were *two* alternative devices for musical reproduction, which, in turn, having one of them come back in vogue, are being challenged in tandem by digital platforms and MP3s). It is why there may be, at any moment, two different prices (costs) on the market, one for each of the technologies marketed as solutions to one problem.

**The Dynamics**

The duopolistic competition comprises successive *cycles* (see figure 1): at time 0, a cycle has just come to an end with firm *a* having won the competition and thus operating at a cost level lower than that of firm *b*. It is the situation at time $-\varepsilon$. At time 0, firm *a*, by virtue of its cumulated *profit* advantage over *b*, is ready to innovate. It is the third fundamental hypothesis.

*Assumption 3:* The firm that has been more profitable at the end of a competitive cycle ends up with sizeable funds — profits — that will be channelled into Research & Development. A firm believes (this is the basic strategic driving *force*) that access to higher technology translates into supremacy on the market; therefore, it will attempt, whenever it can, to set aside sufficient resources wherewith to fund research projects.

*Assumption* 4: The proceeds thus invested will bring about *unfailingly* an innovation, which is then immediately launched on the market: we assume our agents to be powerful trusts bent on exploiting mass-produced innovations to secure corporate success. It is a heroic assumption: of

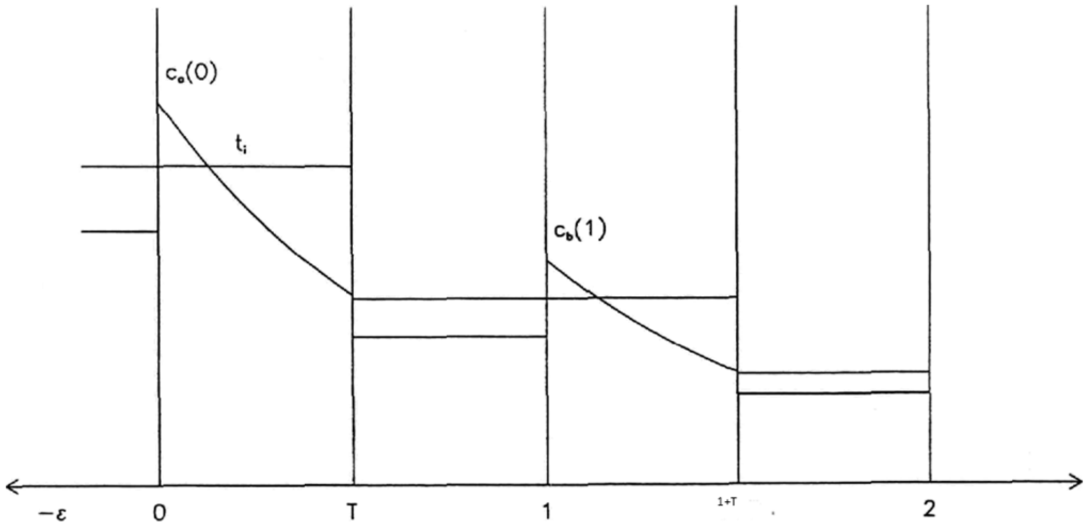


*Fig. 1.* **Duopolistic competition over a cycle***

*Source:* The authors.

* At time $-\varepsilon$, firm *a* has won the competition by reaching a lower average cost of production than *b*'s, as can be seen from the distance between the two segments which respectively represent constant unit costs of production. At time 0, firm *a* innovates and follows thereby a new technological pattern (the learning curve), whereas *b* is "locked-in" its old productive routine. Firm *a*'s new cost level — burdened by R&D expenses — is now higher than *b*'s. As time elapses, however, firm *a*, owing to "learning economies," is able to reduce unit costs: at $t_i$, they are level with *b*'s and beyond that point, *a* becomes more efficient. Meanwhile, firm *b* loses its market share to *a* but has nevertheless the opportunity to "watch and learn" the technical achievements of the adversary. Eventually, at time *T* — when the learning process of firm *a* has plateaued, as can be evinced from the flat portion of the learning curve from *T* onwards — firm *b* comes out with a cheaper copy of *a* s original "combination:" the lower segment from *T* to 1 represents firm *b*'s cost of production for the copy. From *T* to 1, *b* is more competitive and therefore accumulates profit up to a point which will enable it to innovate. This is what takes place at time 1 when *b*'s unit costs make a discrete jump over *a*'s: we have thus obtained exactly the same scenario as the initial one (at time $-\varepsilon$), yet this time around with firm *b* as the innovator and firm *a* as the imitator. The technological strife continues until both firms complete — at time 2 — a full cycle, ending up precisely in the same situation as of time $-\varepsilon$, with *a*'s costs lower than *b*'s; the only difference being the occurrence of two innovations, and the consequent imitations, which have significantly raised since the inception of the competition the overall productive efficiency of the industry (i.e., the average cost of production has been lowered for the industry as a whole).

course, one would like to know exactly why and when certain innovations take place. A model featuring just two firms offers limited options: it must be either firm *a* or firm *b*; and their innovations — for the sake of abstraction — must be forthcoming at regular intervals. The record of the semiconductor industry, which we had in mind while building the model, suggests that this is not an extravagant hypothesis. At any rate, if we take "innovation" to mean an appreciable refinement of a revolutionary idea (and not the revolutionary idea itself: in other words, we are not contemplating a firm which invents fire one day and discovers the wheel the next), then, it is not too far-fetched to make use of this hypothesis considering that what we seek is a crude sketch of industrial dynamics under the pressure of technological change. When the innovation takes place,

*Assumption 5: The costs of the innovator increase discontinuously over those of the rival by a fixed, well-defined amount:* the cost of innovation is reflected by the selling price of the new good. In other words, when first marketed, the innovative solution will be more expensive than the old "way-of-doing- things," on account, i.e., of conspicuous R&D costs.

The competition starts (see figure 2): for a period of time, *b* is more efficient than its rival, which has to bear substantial R&D outlays. But, as time progresses, *a*, which has managed to gain a foothold in a niche market, proceeds along its downward sloping learning-curve, rapidly overtaking its opponent.

*Assumption* 6: After the production cost of the innovator has increased over that of *b* by a constant amount, *a* is capable, by leveraging the learning factor, to reduce the costs of production dramatically: it is now more competitive. While *a innovates,* firm *b* is not experiencing any learning economies: this fact translates into a flat cost curve (see figure 1). At time *T,* firm *a* dominates the market: consequently, its profits are higher

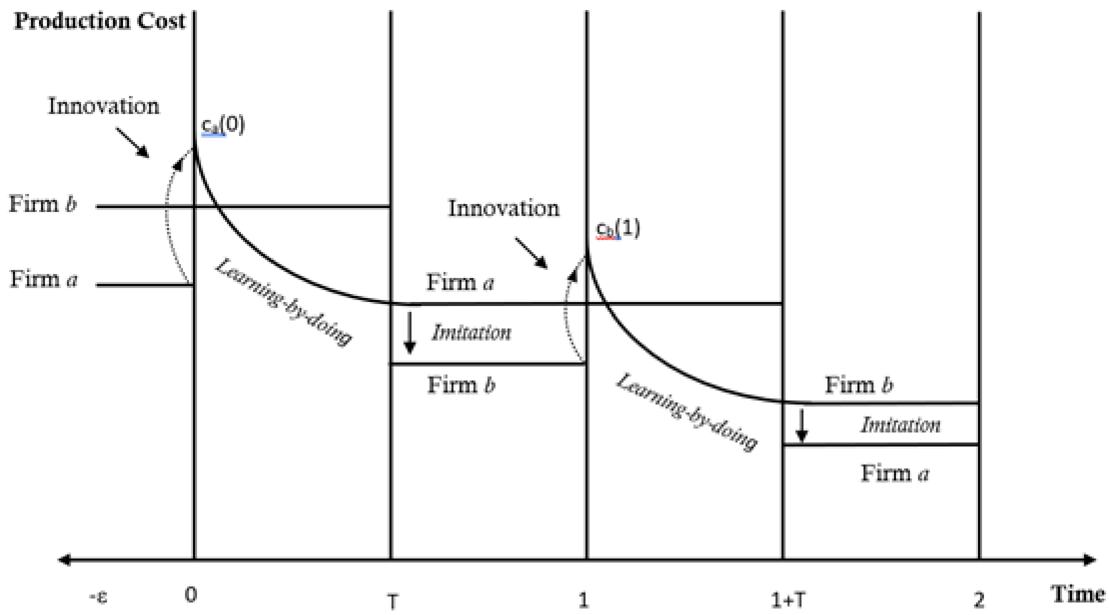


*Fig. 2.* **The complete framework: duopolistic dynamics driven by innovation and imitation**

*Source:* The authors.

than those of firm *b*. Nevertheless, the latter has had the opportunity, between time 0 to *T*-ε, to "watch-and-learn" and is now ready to release a perfect copy of the innovation at a lower price.

*Assumption* 7: When a firm innovates, the other *always* has the opportunity to imitate the original process (we have come to the second fundamental micro-routine, the other being the automatic investment in a novel paradigm) and thereafter offer the cheaper copy for sale. We make the additional hypothesis that *T* — the imitation lag — is a fixed fraction of the innovation period μ (which in this parameterization will be conventionally set equal to 1, a "generic period", so to speak) and that innovation will certainly occur at each μ. Between *T* and 1, the learning curve has reached its region of constancy and the costs of the imitator are lower by a fixed ratio *(B* < 1). If imitation costs are not too high, *b* will earn in the time interval from *T* to 1-ε [that is, (μ-ε)] a volume of profits sufficient to innovate in turn at time 1.

*Assumption* 8 (Restatement of Assumption 3): When the imitator has accumulated a "critical mass" of profits, it is ready to innovate and thus switch role. This development is the joint outcome of Assumption 1 (the entrance of an imitator cannot be prevented), Assumption 3 (profits are invested in R&D) and Assumption 4 (investment in research *automatically* leads to innovation). Strategy-wise, the question at this juncture is: "why would a firm take the trouble to innovate if it foresees that it will be imitated and eventually defeated?" The answer to this question draws on the previous assumptions: we contend that, in a modern credit economy, technology is not so keenly pursued for the enlightened sake of nurturing the soul of humanity by shortening the hours of daily toil, as it is to devise ingenious manufactures or systems with which corporations may wipe out the competition. Ultimately, it is a pecuniary battle that rival clans are waged, so no effort will be spared on their part to devise ever more creative ways of cutting down costs. Thus, according to Assumption 8, agents believe at all times that innovating is preferable to waiting and worth the risk. This deterministic and simplifying hypothesis does not imply that we are ignoring the cases of those firms that have retained for a long time, and profitably so, the role of imitators. These are important instances, yet we think that they do not represent a stable industrial configuration and that, in general, firms have systematically come to consider technology as (one of) the most trenchant weapons in the spasmodic quest for mono- or oligopolistic rent.

From time 1 on (when the imitator turns into an innovator), the foregoing context repeats itself with *b* acting as the leader and *a* as the copycat follower. The former innovates, wins, is imitated and then defeated: another cycle is closed with firm *a* the winner, just as had happened at time -ε, but with a cost level lower than in the previous cycle, for *in the time internal [0,2] two successive innovations occurred*, bringing down the cost level "two notches." As mentioned previously, the party innovating is the one with the higher profits at the end of two subperiods (viz., innovation by one firm followed by imitation by the other). The complete framework is depicted in figure 2.

*Assumption* 9: the *instantaneous* demand expressed in monetary terms $Y$ is constant in time. Formally,

$$Y = c_a(t)x_a(t) + c_b(t)x_b(t). \tag{1}$$

In other words, $Y$ is the total amount of money consumers decide to spend in each instant of time to satisfy their demand. It is constant by assumption. As mentioned above, inertial factors (such as brand loyalty, the sluggish diffusion of information, and long-term binding contracts) prevent consumers, as a *whole,* from buying the cheaper alternative forthwith, thereby eliminating the less efficient producer outright. The point here is to assess *how long* a price difference may persist between the two competing products. Obviously, a price differential between two (somehow imperfect) substitutes cannot obtain for an indefinitely long time: one of the products must eventually fall away.

The further implicit assumption — and it is a strong one — inserted at this juncture is that each phase of the cycle is short enough to allow such price differentials. In our setting, innovation is followed by imitation, which, in turn, is followed by another innovation, etc.: it is the buoyancy of aggressive entrepreneurship that warrants the existence and (temporary) persistence of price differentials. Now, since there are two available competing technologies on the market (good $a$ and good $b$), $Y$ is accordingly divided into two components: the flow of sales of good $a$ and the flow of sales of good $b$.

Indeed, the demand relates to *prices* rather than *costs,* but since we are assuming *constant markups,* the competition, is, in the final analysis, governed by the cost level of each competitor: consequently, $Y$, too, should be divided by the (constant) markup $m$, which, being inessential, shall be omitted. Once again, technological progress drives the competition, not the strategic variables: the technological struggle is the strategic variable *par excellence*, and all tactical considerations entailed by this dynamic duopolistic duel have been subsumed, that is, "hidden," in the notion of "learning-by-doing." In general terms, the best a firm can do is to invest, promote research and hope to come out with an innovative combination that will allow it to survive; if it has not been able to innovate, second best would be to "see" what others have done, learn, imitate —which also spells survival— and, later, try to innovate.

Again, the technological schema we are devising is an extreme simplification of the industrial realm. Notwithstanding the model's terse frame-up, condensing managerial and engineering routines into simple concepts, such as learning and the merit of imitating & borrowing (which translates into a simple per cent discount of the innovator's price), one may still derive a satisfactory description of an important dimension of modern industrial dynamics. Thereby the human factor and the modelling challenges associated with it have all been bypassed; what remains is a macro-dynamic force — i.e., the thrust of technological invention — which is governed by laws of its own that are different from those associated with the microeconomic nature of industrial routines. The model says almost nothing of these microeconomic routines: they are given, and the learning curve represents their cumulated impact; firms leverage these curves to compete on the market; and the global outcome of this prolonged technological strife is the macro-economic development of the industry as a whole. We will draw the wider picture in the last section of the paper. We now turn to the description of the model's dynamics.

From Dosi's model, we retain: 1) the constant elasticity demand function:

$$x_t = A(mc_t)^{-\gamma}, \tag{2}$$

where $\gamma$ is the elasticity; and, in a slightly modified form, 2) the notion of "learning-by-doing" —the larger the production volume, the lower the cost:

$$c_a(t) = c_a(0)\left(\frac{W}{W + \int_0^t x_a dt'}\right)^{\beta}, \tag{3}$$

where

$$X(t) = \int_0^t x_t dt' \tag{4}$$

is the cumulated production; $x_t$ is the flow of demand: the latter can be interpreted as the *selling intensity* of the good demanded (the concept is akin to that of speed: *X(t)* would then correspond to the notion of *distance*). The parameter *W* corresponds to *X(0)*: i.e., the market share of the innovator at time 0. It is now misleading to think that there is such a thing as an *initial* market share when the new product fist reaches the market. We thus prefer to call this quantity *W* and consider it a parameter which, together with β, affects the slope of the learning curve. Alternatively, it can be thought of as that cumulative market share necessary, e.g., when β = 1, to cut the initial cost level in half. By incrementing production, firms have the opportunity to develop the original idea, improve productive techniques, and thereafter abate production costs. In Eq. (3), β is the learning coefficient (the scale of this number will be determined by the chosen parameterization).

*Assumption 10:* A fundamental aspect of the model's dynamics is the apportionment of demand between the two firms. As set down earlier, neither *a* nor *b* can supply the whole market alone. *There is a single demand function (one need) and two producers (two goods):* our *solution is to have the producer of the new good (the innovator at time 0 and the imitator at* time *T) face demand alone, whereas the rival picks up the residual demand,* in keeping with the constraint of Eq. (1).

In other words, in order to determine the market share for each firm, we first use the demand function [Eq. (2)] to "measure" the reaction of consumers to the latest "arrival" on the market and allot the remainder to the incumbent. *Novelty,* not *competitiveness,* determines who faces demand alone. This unorthodox treatment of demand allows us to bypass the difficulty posed by the need to have, at all times, two market shares, two prices (costs), and one demand function.

Once the flow of demand for the new good is derived from Eq. (2) — given the cost levels of *a* and *b* — the flow of demand for the rival is obtained from Eq. (l), which is the expression for the constant outlay *Y* paid by consumers for a given service. By following this method, one can account for the existence of niche markets, such as emerge, e.g., around novel "boutique" items. Despite their expensiveness, they attract a following among more daring, as well as more affluent customers, who can cavalierly afford to "defy" traditional, less efficient, and less sophisticated devices performing the same tasks for a much lower price-tag. Given the potentialities and peculiarities of the consumer habitat, new, exotic products, once they have "landed" on this humus, may very well diffuse and eventually become the new standard.

It is important to bear in mind that in this setting, firms do not set prices as if in some decisional game: instead, they continually refer to their average cost curve, which is driven by the progress of technical learning; they record the level of efficiency achieved and channel that information (plus the mark-up) into the retailing price. This is how prices are determined, and Eq. (2) informs the innovator of his product's impact, while the share apportioned to the other producer is obtained by subtracting from the total demand whatever amount of sales has been grossed by the newly-launched innovation. The model can be interchangeably applied to four possible scenarios:

The endless battle in which the same two firms, *a* and *b,* periodically swap roles (of innovator and imitator): the discussion following the description of the model will be devoted to finding the parameters that warrant this periodical "innovative-attack-and-imitative-reprisal" scenario.

Monopoly under constant attack: the innovator, being more profitable than the imitator in both phases (the innovation phase — from 0 to *T* — and the imitation phase — from *T* to l) succeeds in innovating twice in a row (at 0 and at 1) and thus eliminate the opponent. The latter would be readily replaced by a new entrant eager to challenge the triumphant pioneer at time *T + 1*, and, yet, fated to losing like its predecessor if the parameters characterizing the technological panoply of the innovator and of all other foreseeable imitators[1] do not change.

3) "Sacrifice and the blind cycle of innovation": in this scenario, the innovator suffers the sacrificial fate of the pioneering inventor who ends up immolating himself on the altar of astronomical R&D expenses while a gaggle of industrial plagiarizers ("the imitating collective") steal his idea and successfully recycle it "on the cheap." This is the cautionary apothegm of the "inventor" who is

[1] Viz., steepness of the learning curve, long imitation time lag, prohibitive imitation costs, manageable R&D costs.

fated to wearing a crown of thorns ("*La couronne du novateur, est comme celle du Christ, une couronne d'épines*," Geoffroy de Saint-Hilaire). The story continues with the best-selling imitator, who, after winning the first two rounds (by beating the innovator both in the innovation phase and in the imitation phase), is himself irresistibly and tragically seduced by the siren of creative entrepreneurship. Caught in the trappings of the technological mania, he thus profligately innovates only to be scavenged by another cohort of resourceful imitators, suffering thereby the same commercial death as his predecessor's.

*Ex-Aequo*: It may happen that after a generic innovation/imitation period, the two competitors have earned a comparable amount of profits; in this case, each faces the same probability of innovating: competition proceeds according to the same pattern but in a stochastic fashion. In any event, the long-run dynamics of costs will not differ from the deterministic case, in which firm *a* and firm *b* regularly trade places with each other in leading and following at the end of each period. The discussion of the model's dynamics will ignore this fourth scenario.

It is important to stress once more that all four scenarios feature the interaction of two firms, so that the elimination of a contender does not imply the monopolistic supremacy of the winner: the loser is immediately replaced.

### 4.1. The situation at the end of the previous cycle

Let us recall that a cycle comprises four phases. In the general case, which starts at time $t = 0$, *a* innovates in the first phase, *b* imitates in the second and, provided *b*'s profits exceed those of *a*, it goes on to innovate in the third and is imitated in turn by *a* in the fourth. Such is the snapshot of the game at time -ε (just before *a* innovates): in the last phase of the previous cycle, *a* had become the imitator ($c_a < c_b$): assuming *a grossed* a larger sales volume than *b*'s, it is then ready to innovate. The analysis begins here.

At time -ε, we can normalize the system by setting conventionally the values of 1) the cost level of *a*, 2) the cost level of *b*, lower than that of *a* by a fixed percentage, 3) the intensity of demand for *a* and 4) the intensity of demand for *b*:

$$c_b = 1 \quad x_b = 1$$

$$c_a = B \quad x_a = r, \text{ where } B<1.$$

This implies that

$$Y = 1 + rB. \tag{5}$$

in Eq. (5), $rB$ is, according to Eq. (1), $c_a(-\varepsilon)x_a(-\varepsilon)$, that is, the intensity of total sales netted by *a*, the forthcoming innovator.

### 4.2. The competition in the first phase

The first phase unfolds from 0 to *T*. The innovation triggers a new learning curve whose initial value increases discontinuously by $\frac{1}{B}$ over the cost level of the adversary (set equal to 1). One has then

$$c_a(0) = c_a = \frac{1}{B}.$$

We now come to one of the centrepieces of the model's dynamics. The intensity of demand (for the firm *a*), at time 0 —i.e., *when the innovative product is first "dropped" on the market*— is

$$x_a(0) = x_a = r\left(B^2\right)^{\gamma}.$$

This is the ratio of the flux of demand, Eq. (2), at time 0 to the instantaneous demand at time -ε: to calculate the flow of demand as a function of the average cost of production for any firm, we always compute it at two different (critical) times and take the ratio. It amounts to the normalization of $x_t$ with respect to itself, evaluated at the *preceding stage;* i.e., *x* and its associated cost level are considered at two consecutive periods. By taking the ratio of two consecutive values of *x*, the corresponding (inverse) relationship between the cost levels will tell whether the quantity demanded has increased or not: if *c* (in this time interval) decreases, *x* increases.

In general, this method warrants that the flow of demand for a firm will not increase if its cost level does not change between two successive phases. *Therefore, we never use Eq. (2) as a function of the absolute cost level*;[2] instead, our purpose

[2] Obviously, values of $c_t$ close to zero, which make $x_t$ tend to infinity would render the model totally inconsistent. We thus resort to the normalization of the flux of demand.

is to calculate the intensity of demand for two consecutive periods so as to express the *relative* increase of $x_t$ (relative, i.e., to what it was in the previous phase, given the evolution of the cost structure).[3]

The learning curve is:

$$c_a(t) = \frac{1}{B}\left(\frac{W}{X_a(t)+W}\right)^{\beta}. \qquad (6)$$

By combining the demand schedule with the learning curve, we may write

$$x_a(t) = rB^{2\gamma}\left(\frac{X_a(t)+W}{W}\right)^{\beta\gamma}.$$

By exploiting the relationship between the flux $x_t$, and the cumulative production, [Eq. (4)], one obtains the following differential equation:

$$\frac{dX_a(t)}{dt} = rB^{2\gamma}\left(\frac{X_a(t)+W}{W}\right)^{\beta\gamma}.$$

If $\rho = \frac{X(t)}{W}$, then $d\rho = \frac{dW(t)}{W}$.

We thus have

$$\frac{d\rho}{dt} = \frac{1}{W}\frac{dX_a(t)}{dt} = \frac{r}{W}\left(\frac{X_a(t)+W}{W}\right)^{\beta\gamma} B^{2\gamma},$$

$$\frac{d\rho}{dt} = \frac{r}{W}B^{2\gamma}(1+\rho)^{\beta\gamma}.$$

By separating the variables, the first-order differential equation is easily solved:

$$(1+\rho)^{-\beta\gamma}\, d\rho = dt\,\frac{r}{W}B^{2\gamma},$$

$$\frac{(1+\rho)^{1-\beta\gamma}}{1-\beta\gamma} = t\,\frac{x_a(-\varepsilon)}{W}B^{2\gamma} + C,$$

and by setting the initial condition ρ (0) = 0, one has

$$(1+\rho)^{1-\beta\gamma} = t\,\frac{x_a(-\varepsilon)}{W}(1-\beta\gamma)B^{2\gamma} + 1.$$

[3] One of the implications of such a particular theory of demand is that *A* —the coefficient accounting for the size of the market in Dosi's model — no longer plays this role; here, it is used as a proportional constant whose value, given the dynamics of $x_t$, and $c_t$, may vary from one phase to the next. Yet, this point is of no significance for *A* always cancels out in the derivation of the market share, as shown above.

Using logarithms,

$$(1-\beta\gamma)\ln(1+\rho) = \ln\left[1 + t\,\frac{x_a(-\varepsilon)}{W}(1-\beta\gamma)B^{2\gamma}\right].$$

$$(1+\rho) = 1 + t\,\frac{x_a(-\varepsilon)}{W} = \left[1 + t\,\frac{x_a(-\varepsilon)}{W}(1-\beta\gamma)B^{2\gamma}\right]^{\frac{1}{1-\beta\gamma}}.$$

For 1 − βγ << 1, the previous expression may be reasonably simplified by setting $\frac{1}{1-\beta\gamma} = N$, and using the well-known formula

$$\lim_{N\to\infty}\left(1+\frac{t\alpha}{N}\right)^{N} = e^{\alpha t}.$$

Having set $= \frac{r}{N}B^{2\gamma}$, one easily derives the following exponential trend:

$$X_a(t) = W\left(e^{\alpha t} - 1\right).$$

This relationship expresses effectively the "penetration process" of the new product whose market share, starting from a niche position, increases progressively, indeed exponentially. This relationship may be otherwise stated in terms of flux (i.e., by taking the derivative):

$$x_a(t) = rB^2 \exp\frac{rB^{2\gamma}}{W}t.$$

To sum up: by combining the constant elasticity demand function, which measures the market reaction to the new good, with the learning-by-doing expression (the more a firm produces, the more competitive it becomes), we derive the temporal expansion path of the innovator's market share: the demand function, to repeat, was here used only to determine how the "selling intensity" $x_t$, (for the firm *a*) varied as the cost of production was lowered; the share of *b* is obtained as the difference between total instantaneous expenditures *Y* and the flux of demand for *a* yielded by the foregoing derivation. One may then find the functional nature of the learning curve by substituting in it the preceding expression:

$$c_a(t) = \frac{1}{B}\exp\left(-\frac{1}{}\frac{rB^{2\gamma}}{W}t\right). \qquad \textbf{(7)}$$

As pointed out earlier with Eq. (1), the intensity of demand for firm *b* at time *T* equals the total monetary demand *Y*, minus the flux of demand for the firm *a*, calculated at the same date. Formally,

$$x_b(T) = 1 + rB - rB^{2\gamma-1} \exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T,$$

which, if one sets

$$\lambda = \exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T, \text{ becomes,} \qquad (8)$$

$$x_b(T) = 1 + rB - rB^{2\gamma-1}\lambda.$$

By recalling that $c_b = 1$ and that the markup *m* is constant, one may now determine the profits of *a* and those of *b* over the first phase of the "chase." The formula for the firm *a* is:

$$\Pi_a^{(I)} = (m-1)\int_0^T x_{a(t)} c_a(t)\,dt.$$

Once more, as for the equation of *Y* [Eq. (1)], we shall omit the (irrelevant) "contribution" — (*m* − 1) — of the markup.

$$\Pi_a^{(I)} = rB^{2\gamma-1}\int_0^T \exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)t\,dt =$$

$$= rB^{2\gamma-1}\frac{W}{rB^{2\gamma-1}\left(1-\frac{1}{\gamma}\right)}\left[\exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T - 1\right] =$$

$$= rB^{2\gamma-1}T\left(\frac{\lambda-1}{ln\lambda}\right).$$

The profits earned by *b* are

$$\Pi_b^{(I)} = Y - \Pi_a^{(I)}.$$

If we indicate with $\delta_{ab}^{(I)}$ the difference between these two quantities, we then have

$$\delta_{ab}^{(I)} = \Pi_a^{(I)} - \Pi_b^{(I)} = \left[2rB^{2\gamma-1}T\left(\frac{\lambda-1}{\ln\lambda}\right) - (1+rB)\right]T.$$

For this scheme to be coherent, it is reasonable to assume that this difference is *positive* in the first phase. However, the comparison and the related discussion are postponed to the end of the second phase, when all the elements necessary to determine the critical values that ensure the periodical inversion of roles are available.

### 4.3. The Second Period

From time *T*, a cheaper copy of the innovative product conceived and introduced by *a* at time 0 is ready to be sold by *b*. During the interval from *T* to 1, *b* is the dominant firm. By hypothesis, the efficiency level of *b* is about $\left(\frac{1}{B}-1\right)\%$ (the same percentage assumed in the first phase) higher than the maximum learning level achieved by the other producer, who in the meantime has reached the constancy region of his cost schedule. Formally

$$c_a = c_a^* = \frac{1}{B}\exp -\frac{rB^{2\gamma}}{W}\frac{T}{\gamma},$$

$$c_b = Bc_a^* = \exp -\frac{rB^{2\gamma}}{W}\frac{T}{\gamma}.$$

As for the innovator, we now calculate the flux of demand for the imitator at time *T*, when the copy is introduced in the market. The demand function, Eq. (2), applies to the producer supplying the *new* product: in this phase, the new product is firm *b*'s copy (of *a*'s innovative solution). For the imitator (firm *b*), we compute the intensity of demand at $T-\varepsilon$ (at which the corresponding cost level is 1) and *T*, and then take the ratio (firm *a*'s flow of demand is determined as the residual):

$$x_b = x_b^*\left(\frac{1}{c_b}\right)^\gamma = x_b^*\exp\frac{rB^{2\gamma}}{W}T.$$

The instantaneous revenue for firm *b* is then

$$x_b c_b = x_b^*\exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T.$$

By recalling the substitution (8),

$$x_b c_b = \lambda\left(1 + rB - rB^{2\gamma-1}\lambda\right). \qquad (9)$$

Therefore, the profits for the two players are

$$\Pi_b^{(II)} = (1-T)\lambda\left(1 + rB - rB^{2\gamma-1}\lambda\right)$$

and

$$\Pi_a^{(II)} = (1-T)\left[(1+rB)(1-\lambda)+rB^{2\gamma-1}\lambda^2\right]. \quad (10)$$

The difference is given by

$$\delta_{ab}^{(II)} = (1-T)\left[(1+rB)(1-\lambda)+2rB^{2\gamma-1}\lambda^2\right].$$

### 4.4. Discussion

Through a convenient parameterization of the model, we should be able to determine those critical values for which the interchange of roles is guaranteed to occur *indefinitely* (first scenario). Indeed, because the last two phases of a cycle are the specular image of the first two, the key parameters that allow the competition to unfold "endlessly" assume a general character.

The process comes to an end when a good's production costs, as a result of the duopoly's competitive pressure, drop to a level so low as to force the industry to discard it entirely and thereupon invest novel resources in that sort of research, so typical of modern times, devoted to shaping consumers' tastes by means of "new combinations." This development had also been contemplated by Alfred Marshall [3], who, very much in the spirit of this model's construction, observed in his *Principles* that "this process may go on as long as the risks which are inseparable from the business do not cause him [the producer] exceptional losses; and if it could endure for a hundred years, he and one or two others like him would divide between them the whole of the branch of industry in which he is engaged. The large scale of their production would put great economies within their reach, and provided they competed to their utmost with one another, the public would derive the chief benefit of these economies, and the price of the commodity would fall very low."

Let us consider the following differences:

$$\delta_{ab}^{(I)} = \left[2rB^{2\gamma-1}T\left(\frac{\lambda-1}{\ln\lambda}\right)-(1+rB)\right]T$$

and

$$\delta_{ab}^{(II)} = (1-T)\left[(1+rB)(1-2\lambda)+2rB^{2\gamma-1}\lambda^2\right].$$

We now have to determine for which values of λ; the first expression is *positive* — higher profits for *a* in the first phase — and the second is *negative* — higher profits for firm *b,* which will leverage them to innovate in turn. Why does this comparison is made to hinge on the variable λ? Let us recall that

$$\lambda = \exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T.$$

Now, if arbitrary values are chosen for *r, B* and γ — all of which play an important role in both profit differentials — $\delta_{ab}^{(I)}$ and $\delta_{ab}^{(II)}$ may be expressed as a function of this one variable alone, $\lambda$. The fact that all three mentioned parameters are in the expression for $\lambda$ does not lead to any complication since the latter variable depends on four parameters, $\lambda = \lambda(B,,T,W)$*;* and this degree of freedom allows us to derive coherently the relationship between the two δ's and $\lambda$. We further assume that:

(i) the instantaneous revenue of *a* at time −ε and that of *b* stand in inverse proportion to the ratio of their associated costs, that is

$$\frac{rB}{1} = rB = \frac{1}{B};$$

(ii) the discontinuous increase (spurred by innovation) or decrease (caused by imitation) over the rival's cost-level is arbitrarily and unvaryingly set at 20 per cent, i.e., $B = 0.8$;

(iii) γ = 2. For values of γ close to 1 —this is one of the most fascinating implications of the model— $\delta_{ab}^{(II)}$ *will always be greater than* zero;[4] this means that the "chase" may continue indefinitely *provided the demand for the good is elastic*;

[4] The case of an elasticity equal to 1 has been ruled out because it makes the second δ ($\delta_{ab}^{(II)}$ i.e., the difference between the profits of firm *a* and firm *b* in the second phase) always positive, which is to say that firm *a* is more profitable than *b* even in the second phase: the imitator never stands a chance of making it, regardless of the sign of $\delta_{ab}^{(I)}$. For γ = 1 (i.e., with $\delta_{ab}^{(II)}$ always positive), there is also a range of values of λ for which $\delta_{ab}^{(I)}$ is negative: this would be the anomalous case in which the innovator is (financially) beaten in the pioneering phase yet "manages" to win in the imitation phase: as this was not one of the scenarios previously contemplated, we chose to ignore it. So we firmed our choice of γ = 2, by which one may derive the critical values of the parameters characterizing the first 3 conventional settings listed above: chiefly, 1) the constant inversion of roles; 2) the continued dominance of an innovator over a cohort of imitators; or 3) the ephemeral victory of an imitator, who irresistibly morphs into a pioneer foredoomed to (financial) failure. In sum, the model's implicit suggestion that the demand ought to elastic could be taken to mean that such elasticity is indeed one of the fundamental preconditions for buoyant, incessant (technological) competition: what are here bought and sold are thus leisure goods, i.e., neither necessities nor luxuries.

(iv) to simplify the algebra, $T$ is set equal to $\frac{1}{2}$, viz., it takes twice as long to innovate as to imitate.[5] We now have

$$\delta_{ab}^{(I)} = \frac{1}{2}\left[2B\left(\frac{\lambda-1}{ln\lambda}\right)-\left(1+\frac{1}{B}\right)\right]$$

$$= \frac{1}{2}\left[1.6\left(\frac{\lambda-1}{ln\lambda}\right)-2.25\right].$$

For the second period,

$$\delta_{ab}^{(II)} = \frac{1}{2}\left[(1+rB)(1-2\lambda)+2B\lambda^2\right)] =$$

$$\delta_{ab}^{(II)} = \frac{1}{2}\left[2.25(1-2\lambda)+1.6\lambda^2\right].$$

We are now looking for the *critical* range of λ values which make the "chase" possible. A simple calculation allows us to draw up the following data set:

| $\lambda$ | $\frac{\delta_{ab}^{(I)}}{T}$ | $\frac{\delta_{ab}^{(II)}}{1-T}$ |
|---|---|---|
| 1 | -0.65 | -0.65 |
| 1.4 | -0.34 | -0.91 |
| 1.8 | -0.07 | -0.66 |
| 2.0 | 0.6 | -0.35 |
| 2.2 | 0.18 | 0.09 |
| 2.6 | 0.42 | 1.36 |
| 3.0 | 0.66 | 3.15 |

from which it is easy to see that the sought interval for λ —yielding $\delta_{ab}^{(I)}$ positive and $\delta_{ab}^{(II)}$ negative— is very narrow:

$$1.92 > \lambda > 2.16.$$ [6]

This result may be illustrated graphically (see figure 3) by plotting the expressions for

$$\delta_{ab}^{(I)} = TF^{(I)}(\lambda)$$

and of

$$\delta_{ab}^{(II)} = (1-T)F^{(II)}(\lambda).$$

In sum, for values of $\lambda$ greater than or equal to 2.16, the two δ's are always positive, which means that the innovator is unbeatable, even when challenged by a cheaper copy of his original product; this is the second scenario of the main four contemplated previously. For values of $\lambda$ less than or equal to 1.92 instead (both δ's are negative), the innovator is destined to succumb, prohibitive R&D costs being the harbinger of bankruptcy and consequent "expulsion" from the marketplace: This is the melodramatic scenario (the third) featuring the misadventure(s) of the martyred pioneer, which Schumpeter envisaged in his *Theory of Economic Development.*

It is worth noticing that *W,*[7] *which, along with* β*, determines the shape of the learning curve, is the parameter subject to variation in this case (T, B*

[5] Referring to figures 1 and 2 above, this means that that at time $T$, now set to one-half of the innovation period ($t$ = 1), as $a$'s learning economies plateau for the remainder of the innovation period, *b,* through imitation, discontinuously reduces the price and goes on to win the competition for the rest of the period, i.e., until time $t$ = 1.

[6] Moreover, it is worth noticing that the postulated relationship between revenues and costs [assumption (*i*) of the parameterization, according to which, the instantaneous revenue of *a* and that of *b* are in the inverse ratio as their related costs] still holds at the end of the innovation cycle, i.e., when it is *b*'s turn to create and market a "new combination." Although it surely is a simplifying assumption to take $B$ as a constant throughout the successive phases of the cycle [assumption (ii): $B$ = 0.8], such a choice nevertheless warrants that the competitive configuration (the specific relationship between costs and revenues) prevailing at time $-\varepsilon$ is reproduced at time $1-\varepsilon$, with *b* as the innovator. This may be verified by taking the ratio of both firms' instantaneous revenues at the end of the second period (when the copy is available on the market for the first time), substituting $\frac{1}{B}$ for $rB$ [assumption (i)] and setting it equal to the inverse ratio of their respective costs (for $\gamma$ = 2):

$$\frac{\lambda\left(1+\frac{1}{B}-\lambda B\right)}{\left(1+\frac{1}{B}\right)(1-\lambda)+\lambda^2 B} = \frac{1}{B}.$$

By rearranging terms in the above expressions, one obtains the following quadratic expression in $\lambda$:

$$B(B+1)\lambda^2 - \left(2+\frac{1}{B}+B\right)\lambda + \left(1+\frac{1}{B}\right) = 0,$$

which for $B$ = 0.8, yields $\lambda \cong 2$. We may thus assert that innovation cycles are stationary in the parameter values chosen. In other words, the value of $\lambda$ which preserves the inverse relationship between revenues and costs at the end of the first innovation cycle falls precisely within the critical interval determining the continual inversion of roles. Such a result is indeed an interesting proof of the robustness and consistency of the model.

[7] As was previously discussed, the parameter $W$ corresponds to *X(0)*, i.e., the market share of the innovator at time 0. Since, however, there is no such a thing as an *initial* market share when the new product first reaches the market, it is preferable to regard this quantity as a crucial parameter that affects the slope of the learning curve.

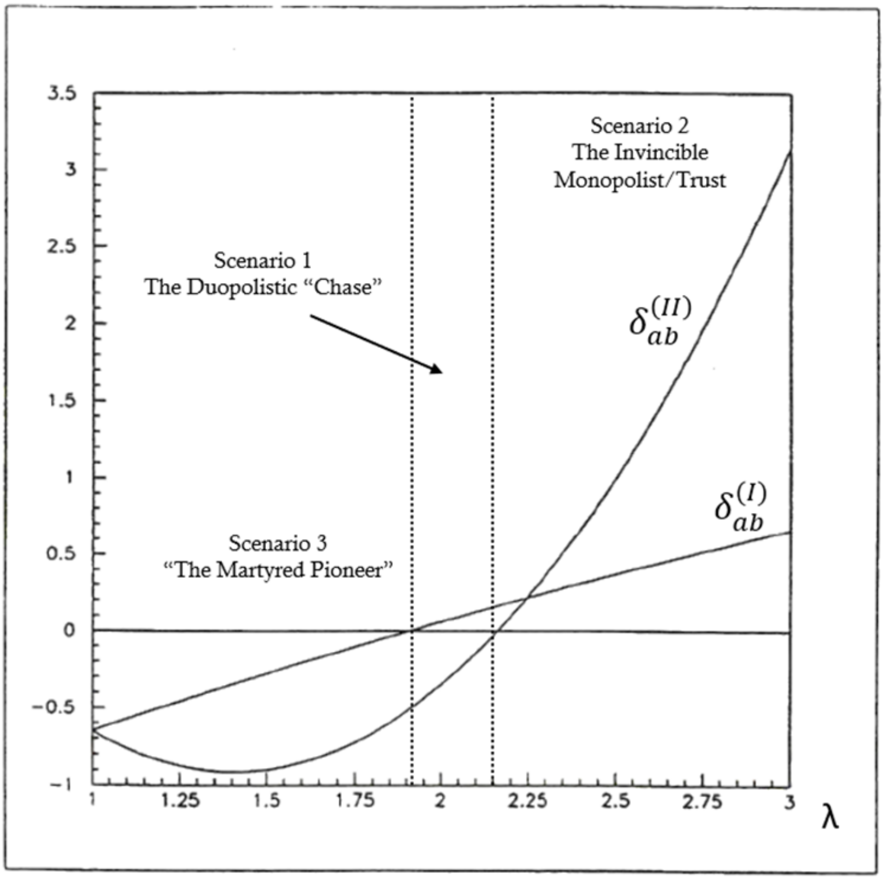


*Fig. 3.* **The determination of the critical range for the interchange of roles**

*Source:* The authors.

and γ have been fixed): the *steeper* the learning curve, the more daunting it is for the imitator to penetrate the market; growing values of *W* reduce the slope of the learning curve (as can be seen by taking the derivative of the learning curve, Eq. (6), with respect to *X*, the cumulative production). In fact, $\lambda$ is a decreasing function of *W*, which implies, again, that the steeper the learning curve, the larger is $\lambda$; and this confirms the contention that high values of $\lambda$ make the innovator unbeatable.

Alternatively, we could have fixed *W* and let *T* (the time lag protecting the innovator) *vary*: obviously, the shorter (longer), the lag, the greater (less) the advantage of the imitator over the incumbent.

Given the critical range for λ and *T* (= 0.5) and keeping in mind that $r = \frac{1}{B^2}$ (sub-hypothesis (*i*)), we may now identify the role of *W*. One has

$$\lambda = \exp\frac{rB^{2\gamma}}{W}\left(1-\frac{1}{\gamma}\right)T = \exp\frac{B^2}{2W}T \cong 2,$$

from which

$$\frac{0.16}{W} \cong ln2 \rightarrow W = 0.23.$$

It may also be of interest to define $t_i$ (see figure 1): starting at time 0, this is the time lapse required by the innovator to reach an efficiency level comparable to that of his competitor *b*, which coincidence occurs when the learning curve, Eq. (8), intersects $c_b = 1$. One may write

$$c_a^{(I)}(t_i) = c_b^{(I)}$$

$$\exp\frac{t_i}{2}\frac{B^2}{W} = B \rightarrow t_i = -\frac{2W}{B^2}lnB = 0.16.$$

Therefore, for this configuration to be stable (scenario 1), the intersection must take place at a time which is about a *third* of the innovation period μ [0, *T*].

### 4.5. The Dynamics of Long-Term Costs

We assume that the threshold $\lambda \cong 2$ is satisfied in each cycle so that *a* and *b* regularly take turns at the helm of the industry, shifting incrementally, and seriatim, the technological frontier.

If, as we have assumed, the innovator's cost regularly decreases with respect to the rival's cost by a fixed amount of 20 per cent, by indicating with 0, 1, 2,... the chronological occurrence of innovative breakthroughs, the sequence of initial cost levels registered at each innovation spike may be written as follows:

$$Time\,0 \to c_a(0) = \frac{c_b(-\varepsilon)}{B} = \frac{1}{B};$$

$$Time\,1 \to c_b(1) = \frac{c_a(1-\varepsilon)}{B} = \frac{c_a^*}{B};$$

$$Time\,2 \to c_a(2) = \frac{c_b(2-\varepsilon)}{B};$$

$$Time\,3 \to c_b(3) = \frac{c_a(3-\varepsilon)}{B};$$

$$and\ so\,on.c_a(T)c_a(1-\varepsilon).$$

The relationship between $c_a(\mathrm{i})$ (or $c_b(\mathrm{i})$) and the successive value is indeed the learning-by-doing cure; in other words, those two values are respectively the initial point ($c_a(0)$ at time 0) and the point of maximum learning $\left(c_a(1-\varepsilon) = c_a^*\right)$ along the same learning curve.

It is now possible to derive the long-run unit cost curve endogenously for the industry as a whole by comparing the two successive cost levels (at time 1 and 0) associated with the two innovations that have taken place during one cycle and then solving a simple recursive relationship (in other words, we are connecting the initial points of the several learning curves which punctuate the competitive path of this hypothetical industry):

$$\frac{c_b(1)}{c_a(0)} = c_a^* = \frac{1}{B\lambda}, \text{since, for} = 2,$$

$$c_a^* = \frac{1}{B}\exp{-}\frac{B^2}{2W}T \text{ and}$$

$$\lambda = \exp\frac{B^2}{2W}T.$$

The solution of the recursive equation is thus

$$c_i = c_0\left(\frac{1}{2B}\right)^i = c_0 e^{i\,ln\left(\frac{1}{2B}\right)}.$$

If one designates with

$$\omega = ln\left(\frac{1}{2B}\right),$$

the parameterization yields the following result: $\omega = 0.47$; and since *i* is the periods' index, in other words, the time *variable,* we finally obtain a unit cost function which, incidentally, happens to coincide with that postulated by Iwai in two important papers[8] on the relationship between industrial structure and technological innovation, i.e.,

$$c(t) = c(0)e^{-\omega t}, \qquad (11)$$

where $\omega$ — which corresponds to $\lambda$ in Iwai's Ansatz — is a normalized parameter for we have set the innovation lag μ equal to 1(see Assumption 7 above).

To recapitulate, this last equation represents the long-run average cost curve for the industry *as a whole:* it is the aggregate outcome of technological strife, fought with innovation-driven onslaughts parried by imitative counterblows. It summarizes a collective process of productive efficiency triggered by the pursuit of profit within the arena of industrial competition.

### 4.6. Summary & Conclusions

In sum, the model suggests inherently that the ingredients warranting buoying competition

[8] The statistical datum reported by Iwai [4, 5] shows how remote the industrial reality has always been from the neoclassical equilibrium picture, according to which, firms characterized by different levels of productivity cannot coexist in the same industry, Graphically, Iwai plots productivity as a function of the number of firms: the relationship assumes the contour of a bell-shaped curve, which is remarkably well approximated by the derivative of a logistic curve — the centerpiece of the model. From the moment a new productive technology is first introduced, this curve describes its adoption path by n firms. In order to obtain a long-run function which unconstrains the relationship between efficiency and the firms' adoption path from the specific timeline of the innovation shifts, Iwai introduces, without accounting for its functional nature, an exponentially decreasing unit cost curve resulting from the continuous improvements brought about by technological development. This allows him to convert the innovation dates to their respective cost levels, i.e., to free the analysis from any further temporal consideration and finally to interpret the relationship thus obtained as a long run industrial configuration. In Iwai's model, the exponential law which portrays the dynamic pattern of costs under the pressure of ingeniousness, is the following: $C(t) = e^{-\lambda t}$, which conveys in simple fashion that creative stimulus tends forces down production costs continuously.

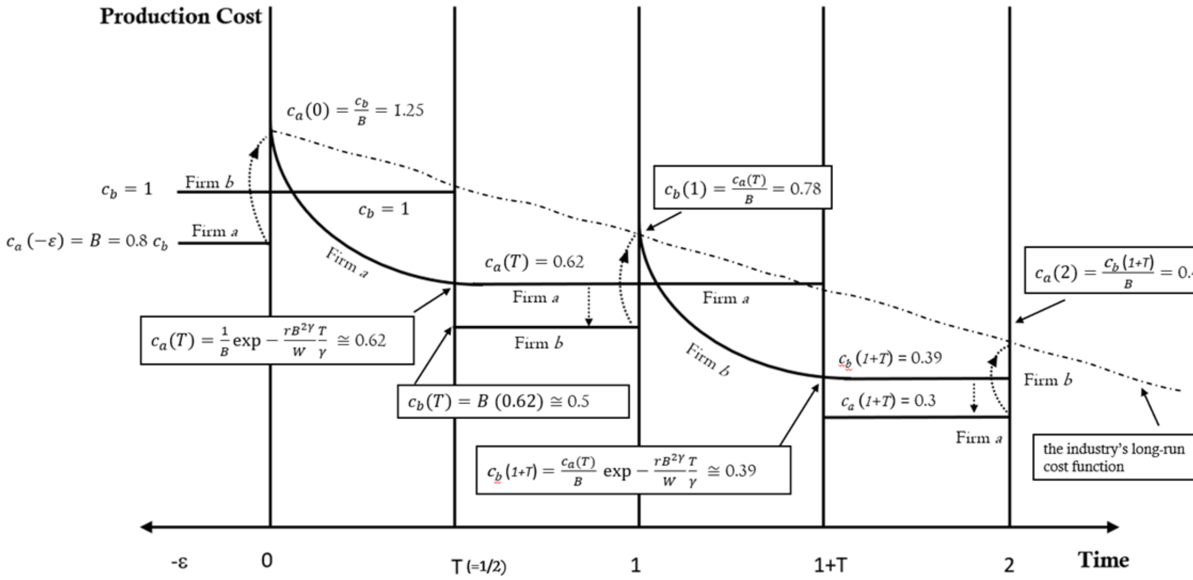


*Fig. 4.* **The long-term derivation of the industry's cost level.***

*Source:* The authors.

* As stated in section 4.2, we do not refer to the absolute level of the intensity of demand, $x_t$, but, rather, normalize this variable (by taking the ratio of its value at a given time to that prevailing in the previous period) with a view to considering exclusively *relative variations*: in this sense, the (slope of) learning curve is not a function of *r* (i.e., the intensity of demand achieved by the imitator prior to innovating). The normalization of $x_a$ ($r$ at $-\varepsilon$, and of $x_b$ at $1-\varepsilon$, etc.) is reset with every innovation to the template of initial values chosen for time $-\varepsilon$. Thus, while maintaining an identical profile, this same learning curve, as shown in figures 1, 2, and especially 4, is merely "pushed down" by ca. $\frac{1}{B\lambda}$ with every innovating click. As far as the model's consistency with regard to the role or *r* is concerned, when $\lambda = 2$ (the "endless chase" scenario), the intensity of demand for firm *b* as it is about to innovate, $x_b(1-\varepsilon) = 2.6$ (calculated via Eq. (9) above; section 4.3), stands to *a*'s intensity of demand at the same time (=1.5, as derived from Eq. (10)) in roughly the same ratio as do the corresponding values at $-\varepsilon$, which is to say, 1.73 (= 2.6 ÷ 1.5) vs. 1.56 $\left(= \frac{1}{B^2} \div 1\right)$.

are: 1) an *elastic* demand (γ = 2); 2) an R&D outlay that is neither prohibitive nor "trifling," i.e., learning economies that are neither too swift (with a steeply declining learning curve, the monopolist/corporate titan always wins) nor too arduous (with a gently downward-sloping learning curve, the innovator is inexorably foredoomed)—the parameter at play here is *W*, by way of λ (λ = 2); and 3), which is a complementary restatement of the previous prerequisite, a lapse time $t_i$ —viz., the time required by the innovator to achieve the same level of efficiency as its rival's— of about *a third* of the innovation period *μ*.

Eq. (11) can also be thought of as encapsulating a "micro-macro" transition phase: that is, a transition from the "micro" to the "macro" dimension. The "micro" sphere is inhabited by the simple economic agents and their routines: here, the learning curves and the imitation responses constitute the "micro" domain. The dynamics of the competition itself and the cumulative result, as portrayed by the long-run average cost curve for the industry, form the "macro"-domain proper of the economic problem at hand. The former delineates the action of *simple* agents. The latter apprehends the mechanics of the system, viewed as a coherent collective of interacting agents.

This model extends *deterministically* the industrial dynamics developed by Iwai: by following the methodological approach of the Neo-Schumpeterian school, according to which the micro-domain is related to the somewhat vague, though the vivid, notion of "routine," competitive settings lend themselves to a more realistic analysis, whose implications are fascinating [6, 7]. No game-theoretic or utility-maximizing concept has been employed here, being the competitive structure completely determined by demand, learning factors and industrial structure (two firms and no significant barrier to entry, save for the imitation lag).

When firms strive to provide the desired service, the strategic variable per se is irrelevant. In this context, the strategic option to manipulate the markup does not seem to affect the long-run configuration: the ineffective producer is simply forced to leave the market. The true strategy itself resides in the organizational and managerial routines, and these cannot be separated from the learning curve. The curve itself, along with an elastic demand function and the chance for newcomers to secure a footing in the market after the innovation, afford, together, a valid description of markets sensitive to technological evolution.

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