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Cryptocurrencies as Volatility Shock Hedges: A Regime-Dependent Fuzzy Copula Analysis of Crypto, Oil, and Equity Markets

P. Mittal
University of Delhi, Delhi, India

ABSTRACT
In the context of increasing integration across global financial markets, understanding the dynamic relationships between energy, digital assets, and equity markets is crucial for effective portfolio management and risk mitigation. **Purpose:** This study examines the potential hedging benefits of cryptocurrencies against volatility shocks by analyzing nonlinear and regime-dependent co-movements among crude oil, Bitcoin, and equity markets. **Method:** The research utilizes a fuzzy copula framework that incorporates Gaussian, Student-t, Clayton, and Gumbel copulas to capture asymmetric and tail-dependent dependence structures. Fuzzy-weighted dependence measures are employed to evaluate dynamic linkages across low-, medium-, and high-volatility regimes, facilitating smooth regime transitions and localized dependence analysis. **Results:** The findings indicate a weak average dependence under normal market conditions, suggesting limited hedging effectiveness of cryptocurrencies. However, statistically significant upper-tail dependence is observed during periods of heightened volatility, indicating a deterioration in diversification benefits under market stress. **Contributions:** The study provides insights for investors, portfolio managers, and policymakers by highlighting the regime-contingent hedging role of cryptocurrencies. The results emphasize the importance of nonlinear and regime-sensitive models in portfolio allocation, risk management, and financial stability assessment.

Keywords: Copula-GARCH; fuzzy logic; co-movement; Bitcoin; oil market; Nifty 50; volatility shocks; regime-switching

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ОРИГИНАЛЬНАЯ СТАТЬЯ

Криптовалюты как средство хеджирования от шоков волатильности: анализ крипторынков, нефтяных рынков и рынков акций с использованием нечеткой копулы

П. Миттал
Университет Дели, Дели, Индия

АННОТАЦИЯ
В условиях растущей интеграции глобальных финансовых рынков понимание динамичных взаимосвязей между энергетическими рынками, рынками цифровых активов и фондовыми рынками имеет решающее значение для эффективного управления портфелем ценных бумаг и снижения рисков. **Цель:** в данном исследовании рассматриваются потенциальные преимущества криптовалют для хеджирования шоков волатильности путем анализа нелинейных и зависящих от режима взаимосвязей между рынками сырой нефти, биткоина и акций. **Метод:** в исследовании используется структура нечеткой копулы, которая включает копулы Гаусса, Стьюдента, Клейтона и Гумбеля для учета асимметричных структур и распределений, зависящих от хвоста. Для оценки динамических связей в условиях низкой, средней и высокой волатильности используются взвешенные по нечетким параметрам меры зависимости, что облегчает плавные переходы

между режимами и локальный анализ зависимости. **Результаты:** результаты показывают слабую среднюю зависимость при нормальных рыночных условиях, что свидетельствует об ограниченной эффективности криптовалют в хеджировании. Однако в периоды повышенной волатильности наблюдается статистически значимая зависимость в верхнем хвосте распределения, указывающая на ухудшение преимуществ диверсификации в условиях рыночного стресса. **Научный вклад:** исследование предоставляет ценную информацию для инвесторов, управляющих портфелями и политиков, подчеркивая роль криптовалют в хеджировании в зависимости от режима. Результаты подчеркивают важность нелинейных и чувствительных к режиму моделей в распределении портфеля, управлении рисками и оценке финансовой стабильности.

Ключевые слова: GARCH-копула; нечеткая логика; совместное движение; биткойн; нефтяной рынок; Nifty 50; шоки волатильности; переключение режимов

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1. Introduction

In an increasingly interconnected global financial system, analyzing the co-movements among crude oil, cryptocurrencies, and stock markets has become essential for understanding cross-asset dynamics and risk propagation [1, 2]. Crude oil continues to serve as a vital determinant of macroeconomic activity, while cryptocurrencies — particularly Bitcoin — represent a decentralized and volatile asset class with growing financialization and institutional adoption. Stock markets, on the other hand, reflect investor sentiment and broader economic performance. Understanding the interdependencies among these heterogeneous assets is crucial for risk diversification, hedging strategies, and policy formulation in the face of shocks such as pandemics, geopolitical conflicts, and energy transitions [3, 4].

From the perspective of emerging economies such as India, this study assumes even greater significance. India's position as one of the world's largest energy importers makes its stock market (Nifty 50 index — the benchmark in the Indian Stock Market.) highly sensitive to global crude oil price movements. Fluctuations in oil prices directly influence inflation expectations, current account balances, and sectoral profitability, especially in energy-intensive industries. Simultaneously, the rapid growth of cryptocurrency adoption in India [5], amid evolving regulatory frameworks, has created a new channel for speculative and cross-border capital flows. These interactions underscore how global energy price volatility and digital asset fluctuations can transmit shocks into the Indian financial system, affecting both domestic market stability and global portfolio diversification patterns. The investigation of co-movement among crude oil, Bitcoin, and Nifty returns therefore provides crucial insights into how India's market dynamics are shaped by global financial forces [6, 7] and, conversely, how emerging market behavior contributes to broader patterns of global financial interconnectedness and systemic risk transmission.

Recent studies have highlighted complex and time-varying connectedness across financial, commodity, and crypto markets, with significant evidence of spillover effects during turbulent periods. For instance, a study by [3] found strong risk spillover between Bitcoin, crude oil, and equity markets during crisis, while [8] revealed energy-stock comovements amplified by pandemic-induced uncertainty. Similarly, [9] documented the causal link between Bitcoin and energy commodities, emphasizing the intertwined nature of digital and physical energy markets. [10] further demonstrated that cryptocurrencies, oil, gold, and US stocks exhibit quantile-dependent volatility interdependencies, suggesting asymmetric responses across different market states. These findings indicate that the direction and strength of co-movement vary over time and depend on prevailing market conditions, such as bullish or bearish regimes [11, 12].

Moreover, [13] underscored the mediating role of oil prices in shaping the relationship between cryptocurrencies and other non-fungible assets, suggesting that crude oil continues to influence the valuation and correlation of emerging digital markets. Similarly, [14] incorporated investor sentiment data from Google Trends to uncover hidden spillovers across financial, crypto, and commodity markets, revealing that sentiment-driven linkages intensify during episodes of global uncertainty. Collectively, these studies confirm that asset interdependencies are nonlinear, asymmetric, and regime-sensitive, underscoring the need for more flexible dependence modeling frameworks.

Traditional approaches — such as linear correlation or GARCH (Generalized Autoregressive Conditional Heteroskedasticity)-based volatility models [4, 15, 16] — are limited in capturing such nonlinear and tail-specific relationships. While correlation coefficients quantify only average dependence, they fail to detect extreme co-movements that occur during market stress. In this context, the copula approach has gained prominence as a robust tool to model joint distributions and dependence structures independently of marginal behaviors [6, 15].

The copula framework has emerged as a robust statistical approach for modeling the dependence structure among multivariate financial variables, as it allows the joint distribution of returns to be decomposed into marginal distributions and a copula function that captures their dependence structure [17]. Copulas enable researchers to identify distinct types of dependencies — such as symmetric, asymmetric, or tail-specific linkages — providing a deeper understanding of how financial assets co-move under normal and extreme market conditions. They are particularly useful in quantifying risk measures such as Value-at-Risk (VaR) and Expected Shortfall, where the choice of copula significantly influences portfolio loss predictions and stress scenario assessments. The copula approach thus provides a flexible tool to simulate extreme events and evaluate portfolio risks beyond the linear correlations typically captured by classical econometric models.

Copula models have been widely applied to understand changing dependence structures in commodity and financial markets. Copula-based models such as Gaussian, Clayton, and Gumbel copulas [18, 19] allow researchers to identify tail dependence, i.e., the likelihood of simultaneous extreme returns across markets, which is critical for risk management and contagion analysis [20].

However, standard copula models treat market states as discrete and static, whereas financial systems are inherently dynamic and uncertain, often exhibiting fuzzy transitions between low, medium, and high volatility phases [21]. The practical application of traditional copulas often encounters possibilistic uncertainty — arising from limited historical data, imprecise parameter estimation, and ambiguity in predicting future market movements. This limitation motivates the use of the fuzzy copula framework, which blends fuzzy logic with traditional copula theory to capture dependence structures that evolve continuously across overlapping market states [22]. By assigning membership weights to different volatility regimes, fuzzy copulas provide a granular view of dependence intensity, accommodating ambiguous boundaries that better reflect real-world market behavior [23, 24]. This innovative approach extends beyond crisp regime classification to reveal localized co-movements and asymmetric linkages that conventional models may overlook.

A study [25] employed a copula-based multivariate GARCH framework to assess the dynamic conditional dependence among various commodity futures markets — including energy, metals, grains, and agricultural commodities — over different market regimes. Their results showed that the dependence structure evolves over time, reflecting regime shifts and market transitions influenced by financialization and investor sentiment. Moreover, they utilized the fuzzy c-means clustering method to classify these market states, highlighting the inherent uncertainty and gradual transitions between volatility regimes rather than abrupt structural breaks.

Recent studies, such as [24], have proposed fuzzy clustering of time series using copula-based lower tail dependence coefficients to identify flexible classification structures in global equity indices. By combining fuzzy entropy regularization with copula dependence measures, such models effectively capture ambiguous, overlapping states of co-movement — a more realistic representation of financial behavior where assets rarely shift between risk regimes in a discrete manner. The fuzzy copula approach therefore enhances traditional dependence modeling by accommodating partial memberships, gradual transitions, and asymmetric tail behaviors within multivariate systems.

Accordingly, the present study introduces a fuzzy copula-based framework to analyze the co-movements among crude oil, Bitcoin, and the Indian stock market (Nifty 50) over the period January 2021 to October 2025. By integrating fuzzy-weighted dependence measures with Gaussian, t, Clayton, and Gumbel copula families, this study aims to uncover nonlinear, tail-dependent, and regime-specific relationships among these asset classes. The findings are expected to offer new insights into portfolio diversification, market integration, and systemic risk under varying market conditions — contributing to the growing literature on cross-asset contagion in the era of digital finance and energy transition.

2. Material and methods

This study investigates the co-movement and dependence structure among crude oil, Bitcoin, and the Indian stock market (Nifty 50) over the period January 2021 to October 2025. Daily closing prices of all three assets were collected, and logarithmic returns were computed as:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \quad (1)$$

where P_t and P_{t-1} represent the price at time t and $t-1$, respectively.

2.1. Preliminary analysis

A preliminary analysis was performed to explore the characteristics of each return series. Descriptive statistics, trend plots, and year-wise box plots were used to visualize dispersion, skewness, and outlier behavior. Stationarity was verified through Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots along with Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The preliminary evidence confirmed that all three-return series were stationary, satisfying the prerequisite for copula-based dependence modeling.

2.2. Copula modeling framework

To model the joint dependence structure, the study applied Copula theory, which allows the joint distribution of random variables to be expressed in terms of their marginal distributions and a dependence function. According to [26], for continuous random variables $X_1, X_2, \ldots, X_n$ with joint distribution $F(x_1, x_2, \ldots, x_n)$ and marginal distributions $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$, there exists a copula C such that:

$$F(x_1, x_2, \ldots, x_n) = C\left(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)\right), \quad (2)$$

where $C:[0,1]^n \to [0,1]$ captures the dependence structure between variables independent of their marginal distributions.

The joint density function can be expressed as:

$$f(x_1, x_2, \ldots, x_n) = c(u_1, u_2, \ldots, u_n)\prod_{i=1}^{n} f_i(x_i), \quad (3)$$

where $u_i = F_i(x_i)$, and $c(u_1, \ldots, u_n)$ is the copula density function.

Four copula families were applied:

- Gaussian Copula — models' symmetric dependence:

$$C_{Gauss}(u, v; \rho) = \Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right), \quad (4)$$

- t-Copula — accounts for symmetric tail dependence:

$$C_t(u, v; \rho, \nu) = t_{\rho,\nu}\left(t_\nu^{-1}(u), t_\nu^{-1}(v)\right), \quad (5)$$

- Clayton Copula — captures lower-tail dependence:

$$C_{Clayton}(u, v; \theta) = \left[\max\left(u^{-\theta} + v^{-\theta} - 1, 0\right)\right]^{-\frac{1}{\theta}}, \theta > 0, \quad (6)$$

- Gumbel Copula — represents upper-tail dependence:

$$C_{Gumbel}(u, v; \theta) = \exp\left\{-[(-\ln u)^\theta + \left(-\ln v\right)^\theta]^{\frac{1}{\theta}}\right\}, \theta \geq 1. \quad (7)$$

Dependence parameters (ρ, θ, ν) were estimated via maximum likelihood estimation (MLE).

2.3. Fuzzy copula framework

While copulas effectively capture nonlinear and tail dependencies, they assume a crisp separation of market regimes. In reality, financial markets transition gradually between states such as low, medium, and high volatility. To address this, the study employed a fuzzy copula framework, integrating fuzzy logic into copula modeling to represent imprecise or overlapping dependence structures.

Each data point is assigned a membership degree $\mu_i(x) \in [0,1]$, representing the extent to which it belongs to a volatility regime i. The fuzzy-weighted copula dependence between two variables X and Y can be represented as:

$$\rho_{Fuzzy} = \frac{\sum_i \mu_i(X)\mu_i(Y)\rho_i}{\sum_i \mu_i(X)\mu_i(Y)}, \quad (8)$$

where ρ_i is the dependence coefficient (e.g., Spearman's ρ or Kendall's τ) within regime i. To measure the divergence between fuzzy and empirical distributions, Kullback–Leibler (KL) divergence was employed:

$$D_{KL}(P \| Q) = \sum_i P_i \ln\left(\frac{P_i}{Q_i}\right). \quad (9)$$

This allows the model to quantify uncertainty in parameter estimation due to limited data or incomplete knowledge of future price behavior. The Fuzzy Copula–GMM (generalized method of moments) framework was used to estimate dependence parameters under uncertain conditions.

2.4. Computational implementation

All analyses were implemented using both Python and R environments. Python libraries — *pandas*, *numpy*, *scipy*, *matplotlib*, *seaborn*, and *openpyxl* — were used for data handling, visualization, and dependence estimation. The R environment (version 4.3) with packages *curl*, *urca*, *tidyverse*, *ggplot2*, *gridExtra*, and *PerformanceAnalytics* facilitated unit root testing, descriptive statistics, and graphical validation.

3. Data analysis and results

3.1. Primary analysis

Figures 1 to 3 provides the trend of Nifty, crude oil, and Bitcoin (BTC) returns during the period from January 2021 to October 2025. The analysis reveals that all three series experienced heightened volatility during 2021–2022, coinciding with the global post-pandemic recovery and geopolitical disruptions. Nifty exhibited moderate fluctuations with an overall upward trajectory, indicating steady market normalization and improving investor confidence through 2023–2025. In contrast, crude oil showed pronounced volatility in 2021–2022 due to supply chain shocks and geopolitical tensions, followed by a period of cyclical stabilization after 2023. Bitcoin, meanwhile, displayed the highest variability among the three, reflecting speculative trading behavior and sensitivity to global regulatory changes. However, its volatility gradually subsided after 2023, signaling a maturing digital asset market. Further, Nifty demonstrated consistent stability, crude oil followed a cyclical pattern tied to external shocks, and Bitcoin underwent a sharp transition from extreme volatility toward relative steadiness. This convergence toward stability across all three markets in recent years suggests improved market integration, reduced uncertainty, and evolving investor behavior across traditional and digital assets.

Table 1 presents the summary statistics of all return series — Nifty, crude oil, and Bitcoin — exhibit near-zero means, indicating the absence of systematic upward or downward trends in their daily returns. Among them, Bitcoin displays the widest range (–0.2593 to 0.1918), reflecting its high volatility and speculative nature compared with traditional assets. Crude oil returns also show large fluctuations, particularly between the first and third quartiles, capturing global price shocks. In contrast, Nifty returns are relatively stable with a narrower dispersion, consistent with its diversified market base.

The Jarque–Bera test results reject normality for all three series ($p < 0.001$), confirming the presence of fat tails and non-Gaussian behavior typical of financial returns. The ARCH LM tests are significant for Nifty and crude oil, and marginally significant for Bitcoin, indicating volatility clustering — periods of alternating high and low volatility over time.

The Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests both confirm that all return series are stationary at level, meaning that shocks to returns are short-lived, and the series revert to their mean over time. The ACF and PACF plots (*Figs. 4 and 5*) further support this result, as all series show rapid decay of autocorrelations and minimal partial autocorrelation beyond lag 1, confirming weak serial dependence.

Overall, these diagnostics validate that the Nifty, crude oil, and Bitcoin return series are stationary but heteroskedastic, making them well-suited for copula-based dependence modeling to capture nonlinear and regime-specific co-movement structures.

Figure 4 shows the autocorrelation structures of Nifty, crude oil, and Bitcoin returns. All three series display rapid decay of autocorrelation values, confirming weak serial dependence and stationarity of the return data. Minor short-term lags are visible in Nifty and crude oil, suggesting limited persistence in daily returns, while Bitcoin exhibits near-zero autocorrelation across all lags, indicating high randomness in its return generation process. This supports the suitability of copula-based modeling, as dependencies arise cross-sectionally rather than temporally. *Figure 5* depicts the partial autocorrelation functions, further confirming that each return series is weakly autocorrelated. The PACF spikes at lag 1 for Nifty and crude oil are marginal but statistically insignificant beyond the first lag, validating the absence of long memory. Bitcoin's PACF remains negligible across all lags, consistent with a random walk pattern. These observations justify the use of copula models focusing on contemporaneous relationships instead of lagged dependencies.

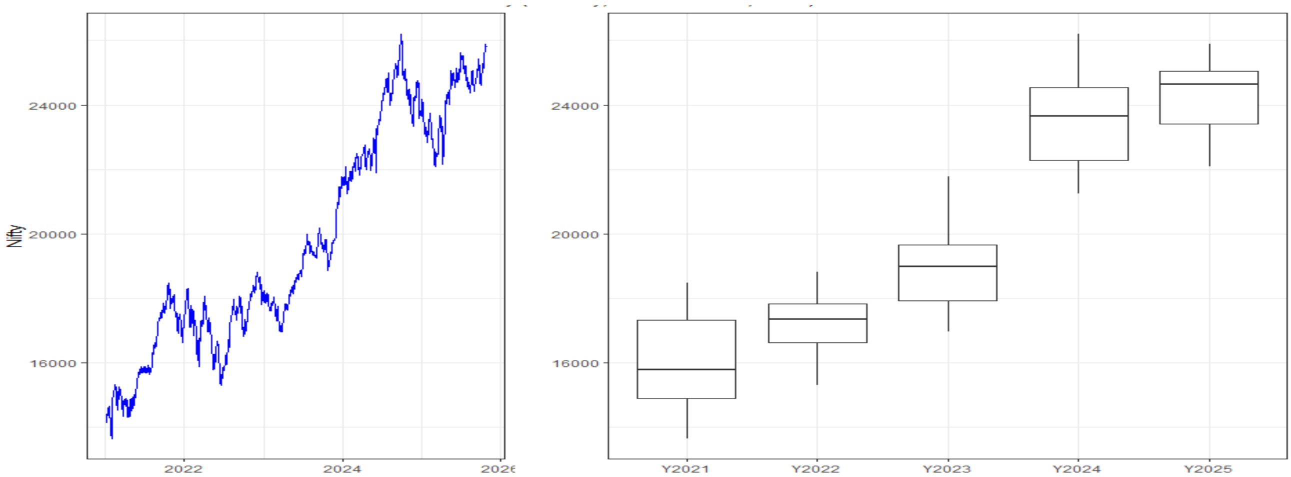

Fig. 1. **Trends and year-wise box plot summary of Nifty (2021–2025)**

Source: Developed by the author using R.

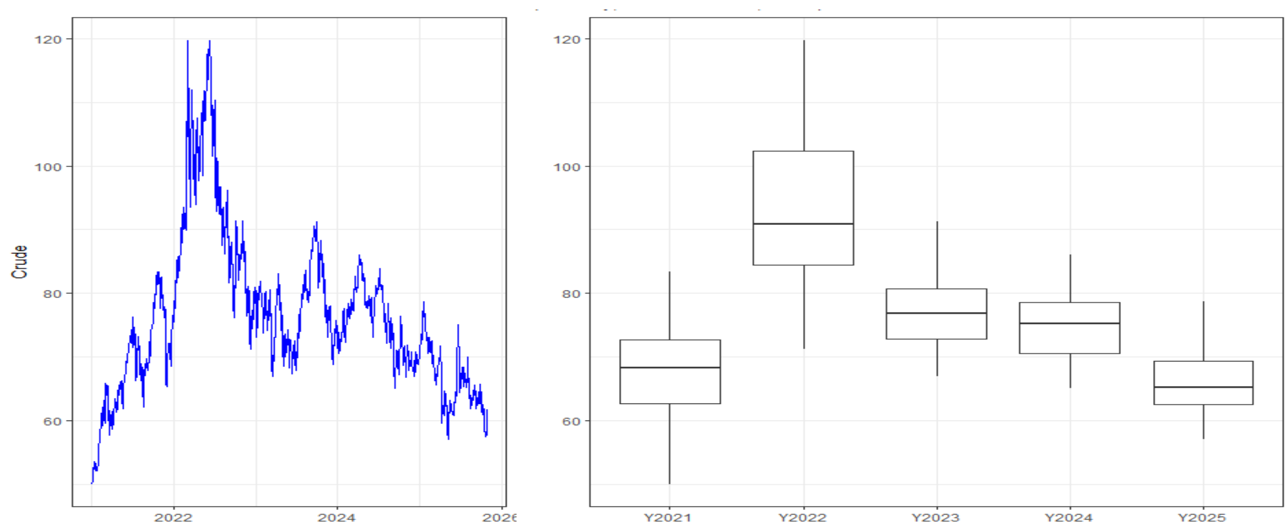

Fig. 2. **Trends and year-wise box plot summary of crude oil (2021–2025)**

Source: Developed by the author using R.

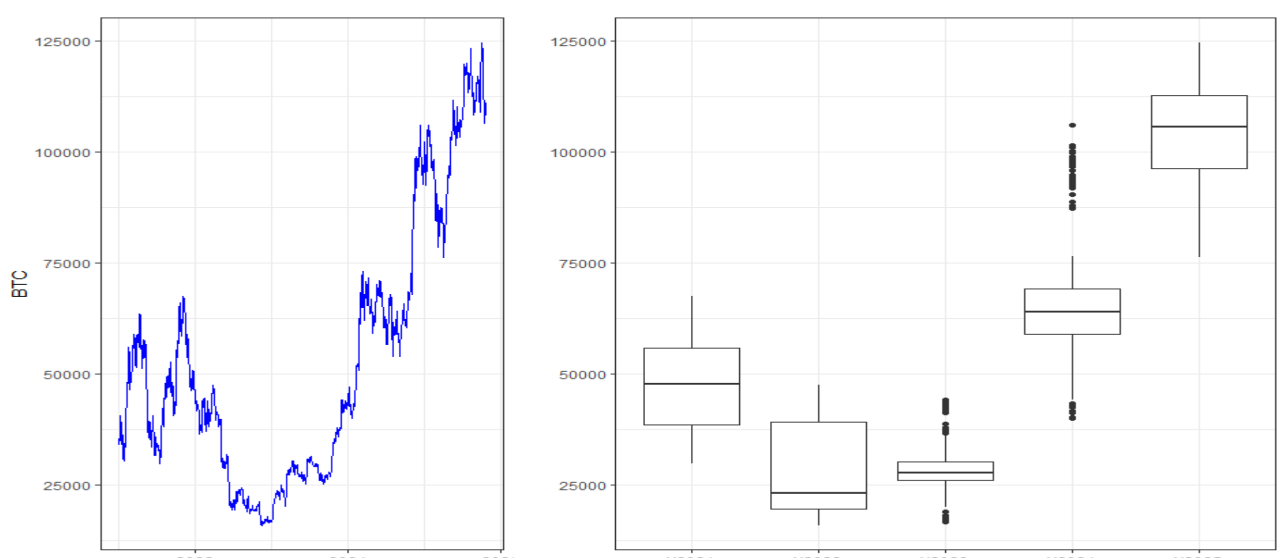

Fig. 3. **Trends and year-wise box plot summary of bitcoin (BTC) (2021–2025)**

Source: Developed by the authors using R.

Table 1

Summary statistics and unit root test results for return series (January 2021–October 2025)

Statistic / Test	Nifty_r	Crude_r	BTC_r
Minimum	−0.0611	−0.1344	−0.2593
1st Quartile (Q1)	−0.0042	−0.0124	−0.0170
Median	0.0007	0.0015	0.0005
Mean	0.0005	0.0002	0.0011
3rd Quartile (Q3)	0.0057	0.0140	0.0191
Maximum	0.0463	0.1354	0.1918
Jarque–Bera (Normality)	923.17 ($p < 0.001$)	785.35 ($p < 0.001$)	1647.3 ($p < 0.001$)
ARCH LM (Heteroskedasticity)	$\chi^2 = 105.78$, p < 0.001	$\chi^2 = 102.41$, p < 0.001	$\chi^2 = 21.53$, p = 0.043
ADF Test (Stationarity)	DF = −33.958, p = 0.01	DF = −34.176, p = 0.01	DF = −35.487, p = 0.01
PP Test (Stationarity)	$Z(\alpha) = -1127.1$, p = 0.01	$Z(\alpha) = -1023.8$, p = 0.01	$Z(\alpha) = -1206.8$, p = 0.01
Conclusion	Stationary with volatility clustering	Stationary with volatility clustering	Stationary with moderate volatility clustering

Source: Author's calculation using R.

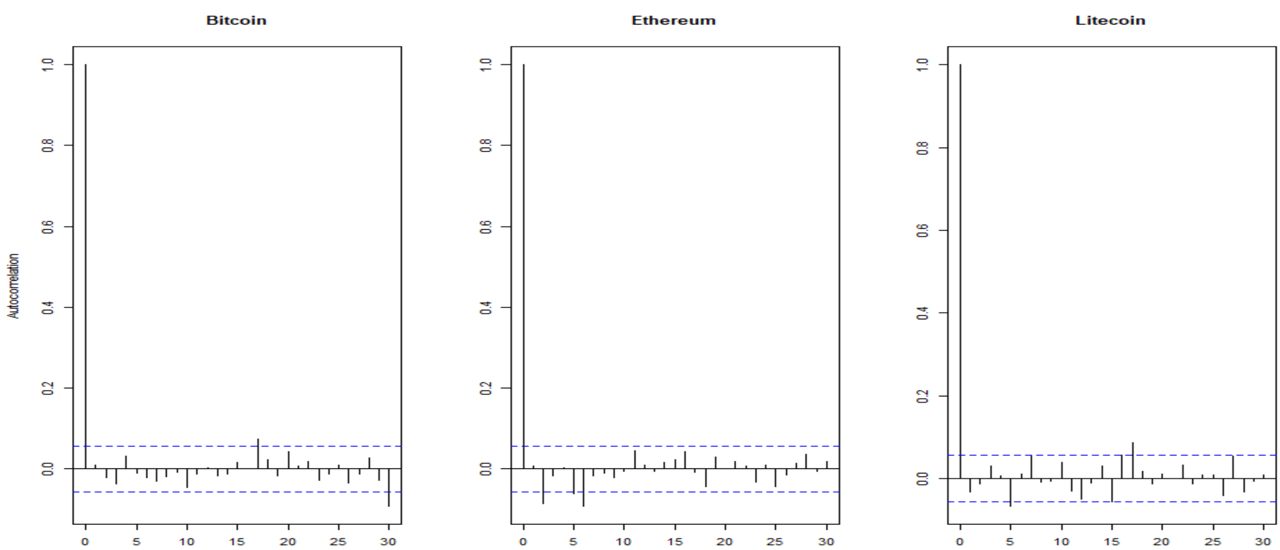

Fig. 4. **Autocorrelation functions (ACF) of Nifty, crude oil and BTC return series**

Source: Developed by the author using R.

Figure 6 visualizes the correlation structure between crude oil and Bitcoin returns, showing a dispersed cloud of data points without any clear linear alignment. This pattern confirms the results of the conventional correlation analysis, which reveals an insignificant linear relationship between the two assets. The absence of a consistent pattern indicates that traditional linear methods fail to capture the true nature of their dependence. However, the presence of localized clusters in certain regions suggests short-lived, regime-specific co-movements during extreme market conditions. These observations highlight the limitation of classical correlation measures and underscore the importance of adopting non-linear dependence models, such as copula and fuzzy-based approaches, which are better suited to identify asymmetric, tail, and state-dependent relationships between heterogeneous assets like crude oil and Bitcoin.

3.2. Dependence between Nifty and crude oil returns

This section presents the dependence structure among the return series of Nifty, crude oil, and

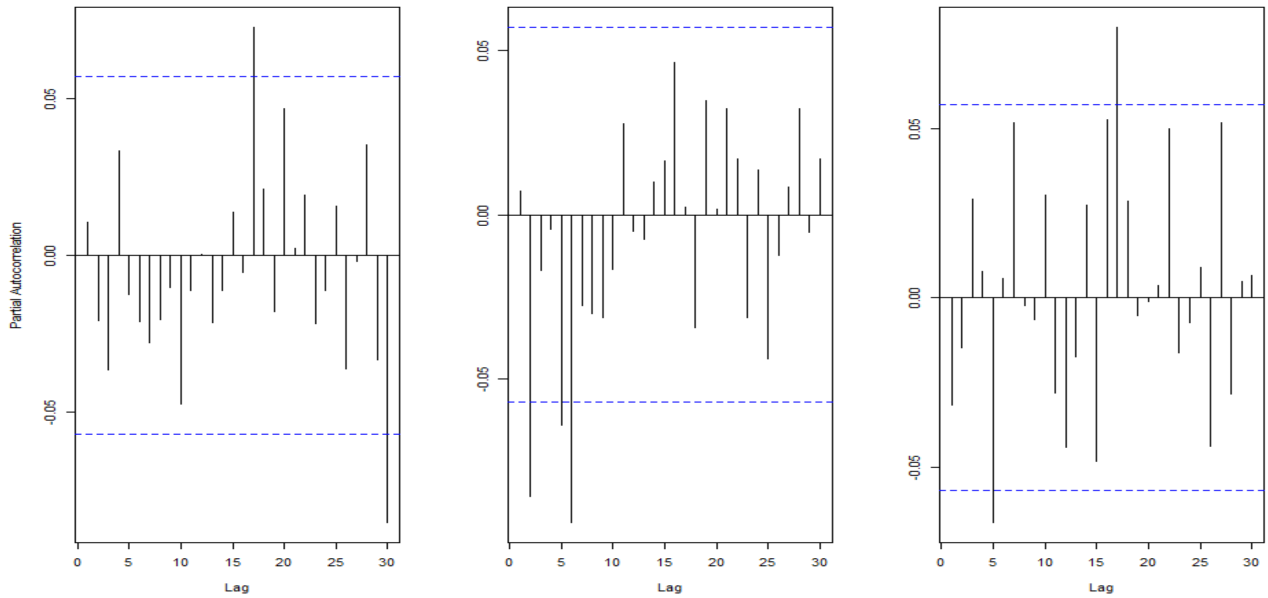

Fig. 5. **Partial autocorrelation functions (PACF) of Nifty, crude oil and BTC return series**

Source: Developed by the author using R.

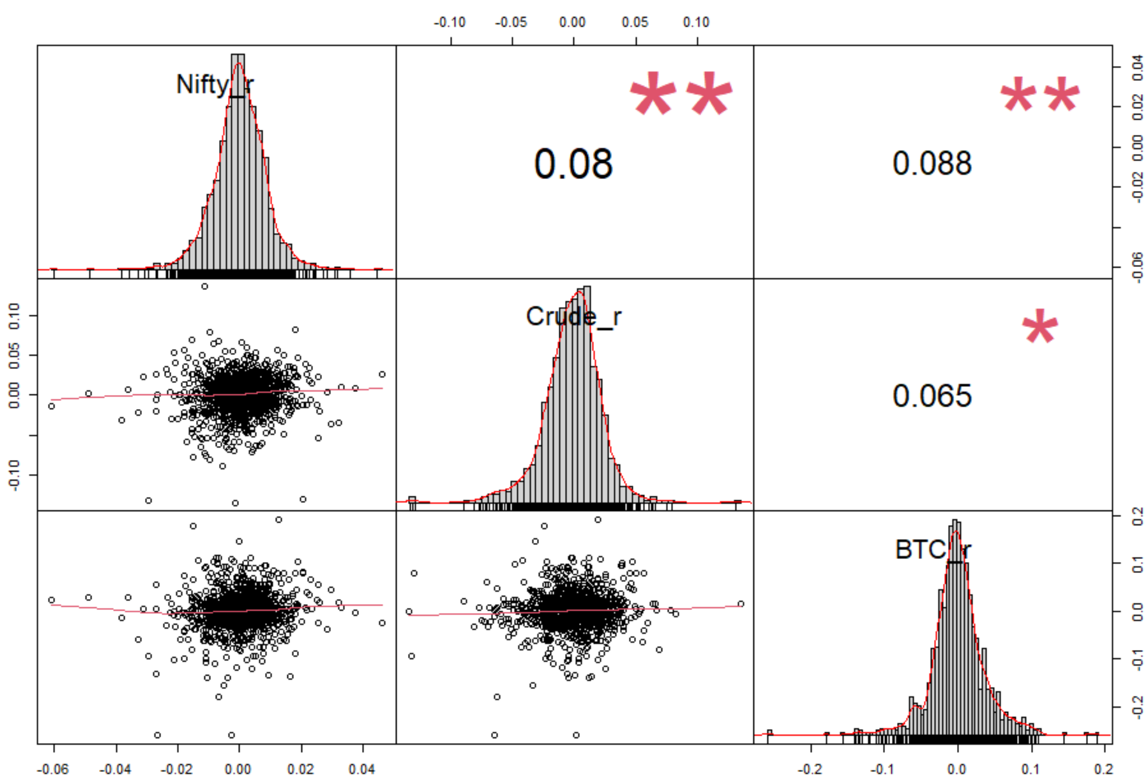

Fig. 6. **Correlation analysis of crude oil and BTC return series**

Source: Developed by the author using R.

Bitcoin (BTC) based on classical rank correlations, Gaussian copula correlations, and fuzzy copula analysis. Pairwise results are discussed with detailed fuzzy-state breakdowns, copula fits, and interpretive remarks.

The results presented in *Table 2* indicate a negligible dependence between Nifty and crude oil returns across multiple correlation measures. The Spearman's rho (ρ = 0.0911, *p* = 0.0017) and Kendall's tau (τ = 0.0625, *p* = 0.0013) reveal a statistically significant yet economically weak monotonic association between crude oil prices and Nifty returns. Similarly, the Gaussian copula correlation (ρ = 0.0851) confirms the presence of a very mild

Table 2

Dependence measures between Nifty and crude oil returns

Measure	Estimate	p-value	Interpretation
Spearman's ρ	0.0911	0.0017	Negligible monotone dependence
Kendall's τ	0.0625	0.0013	Negligible concordance
Gaussian Copula ρ	0.0851	–	Negligible linear dependence (normal scores)

Source: Author's calculation using R.

Table 3

Fuzzy-weighted dependence breakdown between Nifty and crude oil returns

Fuzzy Set Combination	Weighted Spearman	Weighted Kendall-like	Total Weight	Interpretation
Low-Low	0.3428	0.0724	21.54	Moderate positive dependence in lower regime
Low-Medium	–0.0030	–0.0018	62.79	No meaningful dependence
Low-High	–0.4417	–0.1351	17.91	Strong inverse dependence (cross-regime)
Medium-Low	0.1679	0.0916	79.63	Weak positive linkage
Medium-Medium	0.0841	0.0575	968.50	Weak but stable association
Medium-High	–0.0704	–0.0165	70.00	Weak inverse relation
High-Low	–0.2224	0.0142	19.42	Negative cross-regime dependence
High-Medium	0.0651	0.0461	81.72	Slight positive co-movement
High-High	0.3047	0.0675	24.57	Moderate upper-tail dependence

Source: Author's calculation using R.

linear dependence when transformed to normal scores, reinforcing the observation that the relationship between the two markets is largely random with only occasional localized co-movements. This weak dependence highlights the limited spillover effects between the Indian equity index and the global crude oil market during the study period, indicating a limited co-movement between crude oil prices and Nifty returns rather than any causal relationship.

The fuzzy-weighted dependence analysis in *Table 3* reveals a nuanced and regime-specific pattern between Nifty and crude oil returns. In the Low-Low regime, both variables display a moderate positive dependence (Weighted Spearman = 0.3428; Weighted Kendall-like = 0.0724), implying that when both markets are in low-return phases, their movements tend to align positively. The Low-High combination, however, shows a strong inverse dependence (–0.4417; –0.1351), indicating that when crude oil prices rise sharply while Nifty remains low, the relationship turns distinctly negative — reflecting potential cost-push pressures or investor flight from equities during oil price surges.

In the Medium-Medium regime, which dominates with the highest total weight (968.50), the dependence remains weak but stable, suggesting that most market observations lie in this zone with limited but consistent co-movement. The Medium-Low and High-combinations indicate weak positive linkages, pointing to partial co-movement when one market shifts toward stability or moderate performance. Conversely, the Medium-High and High-Low regimes reflect weak to moderate negative dependence, capturing op-

Table 4

Copula model fit statistics for the Nifty-crude oil returns

Copula	Log-Likelihood	Parameter	Interpretation
Gaussian	4.297	—	Weak symmetric dependence
t-Copula	13.917	$\nu = 8$	Slight tail dependence
Clayton	6.483	$\theta = 0.1332$	Weak lower-tail association
Gumbel	435.441	$\theta = 1.0666$	Mild upper-tail dependence (best fit)

Source: Author's calculation using R.

Table 5

Dependence measures between Nifty and Bitcoin returns

Measure	Estimate	p-value	Interpretation
Spearman's ρ	0.0639	0.028	Negligible monotone dependence
Kendall's τ	0.0429	0.027	Negligible concordance
Gaussian Copula ρ	0.0769	—	Negligible linear dependence

Source: Author's calculation using R.

posite movements during transitional or volatility-driven phases.

Finally, the High-High regime exhibits a moderate upper-tail dependence (0.3047; 0.0675), implying that in periods of simultaneous high returns, both Nifty and Crude Oil tend to move together, albeit not strongly. This pattern may reflect periods of strong macroeconomic conditions or global demand expansion, where rising economic activity simultaneously drives equity market performance and increases demand for crude oil, leading to a positive co-movement. In contrast, the inverse relationship observed in the Low-High regime may capture periods of cost-push pressures or market stress, where rising oil prices adversely affect equity returns. These findings suggest that dependence between Nifty and crude oil is asymmetric and largely driven by regime-specific dynamics — stronger at the extremes (low-low and high-high) and weak or inverse in cross-regime transitions influenced by underlying economic conditions.

The results suggest negligible overall dependence between Nifty and crude oil returns, though the fuzzy-weighted analysis uncovers *localized structure*: positive correlation during simultaneous low or high return regimes, and negative dependence across opposite states. As shown in *Table 4*, the Gumbel copula provides the best fit with a log-likelihood of 435.441, indicating mild upper-tail dependence between Nifty and crude oil returns — suggesting that co-movements strengthen during high-return periods. The t-Copula ($\nu = 8$) also captures slight tail dependence, consistent with occasional joint extremes. Both the Gaussian and Clayton copulas show weak dependence structures, with the Clayton indicating limited lower-tail association. The Gumbel copula offers the best fit, indicating mild upper-tail dependence. Overall, the results highlight asymmetric dependence dominated by upper-tail co-movements, aligning with the fuzzy analysis that shows stronger interaction during extreme market phases.

3.3. Dependence between Nifty and Bitcoin returns

Table 5 shows that the overall dependence between Nifty and Bitcoin returns is negligible across all measures. The Spearman's ρ of 0.0639 ($p = 0.028$) and Kendall's τ of 0.0429 ($p = 0.027$) indicate very weak monotonic and concordant relationships, suggesting that the two markets move largely independently. Similarly, the Gaussian Copula correlation of 0.0769 reinforces the absence of any meaningful linear association between their normal-score transformations. These results imply that Bitcoin's price dynamics are largely decoupled from Nifty's performance, re-

Table 6
Fuzzy-weighted dependence breakdown between Nifty and Bitcoin returns

Fuzzy Set Combination	Weighted Spearman	Weighted Kendall-like	Total Weight	Interpretation
Low-Low	0.4205	0.1040	18.52	Strong positive linkage in downturns
Low-Medium	0.0424	0.0327	62.79	Weak positive association
Low-High	−0.2661	−0.0332	17.52	Inverse dependence (cross-regime)
Medium-Low	0.1429	0.0698	58.37	Weak positive co-movement
Medium-Medium	0.0590	0.0387	970.41	Very weak stable association
Medium-High	−0.0033	0.0069	82.66	No meaningful relation
High-Low	−0.2904	−0.0346	16.28	Negative cross-regime dependence
High-Medium	0.0528	0.0236	81.94	Weak positive dependence
High-High	0.3023	0.0378	24.19	Moderate upper-tail dependence

Source: Author's calculation using R.

flecting distinct market structures and investor bases with minimal spillover effects.

The fuzzy-weighted dependence results in *Table 6* reveal that the relationship between Nifty and Bitcoin returns is weak overall but varies across regimes. A strong positive linkage is observed in the Low-Low regime, indicating that both assets tend to move together during downturns, possibly reflecting synchronized risk aversion or liquidity withdrawals. The Low-High and High-Low combinations show inverse dependence, suggesting opposite behavior when one market experiences gains while the other declines. The Medium-Medium regime, which carries the highest weight, exhibits a very weak yet stable association, signifying largely independent performance under normal market conditions. In the High-High zone, a moderate upper-tail dependence is observed, implying that during bullish phases, Bitcoin and Nifty occasionally move in the same direction. Overall, dependence is minimal in normal conditions but becomes more evident during extreme market states.

The classical dependence between Nifty and BTC is weak, but the fuzzy copula breakdown identifies positive co-movement in both *low-low* and *high-high* regimes. The Gumbel copula fits best (log-likelihood = 453.193), suggesting a slight upper-tail dependence between Nifty and Bitcoin returns — indicating limited co-movement during periods of simultaneous market optimism (see *Table 7*). The t-Copula ($\nu = 10$) captures minor tail dependence, consistent with occasional joint extremes, while both the Gaussian and Clayton copulas reflect weak and nearly symmetric dependence structures. Overall, the results confirm that the linkage between Nifty and Bitcoin is generally weak, with mild synchronization price rises in extreme market conditions.

3.4. Dependence between crude oil and bitcoin returns

Table 8 indicates that the relationship between crude oil and Bitcoin returns is statistically insignificant and economically negligible. Both Spearman's ρ (0.0509, $p = 0.080$) and Kendall's τ (0.0343, $p = 0.077$) are small and not significant at the 5% level, suggesting an absence of consistent monotonic or concordant movement between the two assets. Similarly, the Gaussian Copula correlation of 0.0572 reinforces the lack of meaningful linear association. These findings imply that Bitcoin's return dynamics are largely independent of global oil price fluctuations, reflecting the distinct nature and drivers of the cryptocurrency and commodity markets.

The fuzzy-weighted dependence results in *Table 9* show generally weak but regime-dependent

Table 7

Copula model fit statistics for the Nifty–Bitcoin returns

Copula	Log-Likelihood	Parameter	Interpretation
Gaussian	3.509	–	Weak symmetric correlation
t-Copula	8.325	$\nu = 10$	Minor tail dependence
Clayton	5.783	$\theta = 0.0897$	Negligible lower-tail dependence
Gumbel	453.193	$\theta = 1.0448$	Slight upper-tail dependence (best fit)

Source: Author's calculation using R.

Table 8

Dependence measures between crude oil and Bitcoin returns

Measure	Estimate	p-value	Interpretation
Spearman's ρ	0.0509	0.080	Not significant; negligible dependence
Kendall's τ	0.0343	0.077	Not significant
Gaussian Copula ρ	0.0572	–	Negligible linear dependence

Source: Author's calculation using R.

relationships between crude oil and Bitcoin returns. A moderate positive linkage appears in the Low-Low regime, suggesting that both assets may experience mild co-movement during downturns. The Low-High and High-Low regimes reveal negative cross-regime dependence, indicating opposite movements when one market strengthens while the other weakens. The Medium-Medium regime, which dominates with the highest weight, shows minimal association, implying near-independence under typical market conditions. In contrast, the High-High regime exhibits moderate upper-tail dependence, suggesting occasional co-movement during strong bullish periods. Overall, the relationship between crude oil and Bitcoin remains weak, with dependence becoming more evident only at market extremes.

Crude oil and BTC returns exhibit no significant overall correlation. However, fuzzy decomposition highlights positive dependence during joint downturns (*low-low*) and joint upswings (*high-high*). The best-fitting Gumbel copula reveals a weak but consistent upper-tail relationship, suggesting that both assets tend to rise together during bullish phases.

As shown in *Table 10*, the Gumbel copula achieves the highest log-likelihood value (463.440), indicating that the crude oil and Bitcoin return exhibit mild upper-tail dependence, or limited co-movement during bullish market conditions. The t-Copula ($\nu = 20$) captures mild symmetric tail dependence, suggesting occasional simultaneous extreme movements in both directions. The Gaussian and Clayton copulas reflect weak linear and lower-tail associations, respectively, implying minimal correlation under normal conditions. Overall, dependence between crude oil and Bitcoin remains weak but becomes more noticeable during simultaneous high-return periods.

3.5. Comparative summary

Across all pairs, conventional dependence correlation measures are weak, but fuzzy copula results expose *nonlinear and asymmetric co-movements* (see *Table 11*). The consistent dominance of the Gumbel copula indicates an overarching *upper-tail dependence pattern*, implying that these assets may rise concurrently in bullish market extremes.

4. Discussion

The results offer significant insights into the relationships between crude oil, Bitcoin, and the Nifty 50 index, indicating that these connections are not strong when assessed using conventional metrics such as Spearman's ρ and Kendall's τ.

Table 9
Fuzzy-weighted dependence breakdown between crude oil and Bitcoin returns

Fuzzy Set Combination	Weighted Spearman	Weighted Kendall-like	Total Weight	Interpretation
Low-Low	0.3304	0.0543	19.28	Moderate positive relation in downturns
Low-Medium	0.0568	0.0339	79.14	Weak dependence
Low-High	−0.1575	0.0574	19.81	Weak inverse relation
Medium-Low	0.0429	0.0191	57.95	Weak positive linkage
Medium-Medium	0.0470	0.0322	965.20	Minimal association
Medium-High	−0.0207	−0.0103	82.50	Very weak inverse dependence
High-Low	−0.2412	−0.0131	15.48	Negative cross-regime effect
High-Medium	0.0694	0.0404	70.15	Weak positive association
High-High	0.3154	0.0463	21.86	Moderate upper-tail dependence

Source: Author's calculation using R.

Table 10
Copula model fit statistics for the crude oil and Bitcoin returns

Copula	Log-Likelihood	Parameter	Interpretation
Gaussian	1.940	–	Weak linear correlation
t-Copula	2.876	$\nu = 20$	Mild symmetric tail dependence
Clayton	2.737	$\theta = 0.0711$	Weak lower-tail association
Gumbel	463.440	$\theta = 1.0356$	Upper-tail dependence (best fit)

Source: Author's calculation using R.

Table 11
Summary of dependence measures and best-fitting copulas among Nifty, crude oil, and Bitcoin return pairs.

Pair	Spearman's ρ	Kendall's τ	Gaussian ρ	Best-Fitting Copula	Copula Parameter	Dependence Type
Nifty-Crude	0.091	0.063	0.085	Gumbel	$\theta = 1.0666$	Mild upper-tail, regime-specific
Nifty-BTC	0.064	0.043	0.077	Gumbel	$\theta = 1.0448$	Weak upper-tail, localized co-movement
Crude-BTC	0.051	0.034	0.057	Gumbel	$\theta = 1.0356$	Weak upper-tail, simultaneous rallies

Source: Author's calculation using R.

This suggests that the linear and monotonic relationships are limited. This aligns with previous studies, for example [27–29], reporting weak average linkages between traditional financial assets and cryptocurrencies.

However, the copula-based results reveal that dependence is asymmetric and distribution-dependent. The consistent selection of the Gumbel copula across pairs underscores the presence of upper-tail dependence, implying that co-movements become more significant during extreme positive market conditions. This behavior is consistent with findings of [11, 30] on nonlinear and tail-based dependence structures in financial markets.

The findings from the fuzzy copula approach enhance our comprehension by highlighting how dependence varies with different states. Rather than presuming constant relationships, this fuzzy model reveals how dependence shifts across low, medium, and high return scenarios. The analysis indicates that the medium–medium scenario holds the greatest significance, signifying stable yet weak connections during typical market conditions. Conversely, more pronounced dependence-both positive and negative-appears in extreme scenarios, especially in combinations like low-high and high-low. These trends suggest uneven adjustments, where disturbances in one market may not spread uniformly but are influenced by the current state of the other market. This supports the view that financial market relationships are state-dependent and evolve gradually, rather than exhibiting abrupt shifts [31]. Overall, the results suggest that dependence among these markets is episodic and nonlinear, shaped by global shocks and changing investor sentiment, reinforcing the importance of advanced modeling frameworks for capturing such complexities.

5. Conclusions

This study examined the nonlinear dependence and co-movement dynamics among crude oil, Bitcoin, and the Indian stock market (Nifty 50) using a fuzzy copula-based approach. Conventional correlation and linear models revealed weak and statistically insignificant average dependence, indicating that the relationships among these asset classes cannot be adequately captured through standard measures. However, the fuzzy copula framework uncovered distinct regime-specific and tail-dependent linkages, particularly highlighting moderate upper-tail dependence between oil and cryptocurrency returns during periods of extreme volatility.

The fuzzy copula analysis provides a richer picture of dependence among Nifty, crude oil, and Bitcoin returns than conventional correlation metrics. Although the overall dependence appears weak, the fuzzy decomposition highlights *state-dependent relationships* — notably stronger co-movements in extreme low and high regimes. This suggests that market linkages intensify under stress or exuberance, supporting the notion of asymmetric contagion across asset classes. The Gumbel copula's consistent superiority indicates mild upper-tail dependence, implying simultaneous price increases across assets during bullish conditions. From a portfolio management perspective, these findings underscore that traditional diversification benefits may weaken during market rallies, while during normal conditions, interdependence remains minimal. Thus, fuzzy copula models can better capture localized and nonlinear dependence patterns, offering valuable insights for risk diversification, hedging, and stress-testing in multi-asset portfolios.

The findings highlight the critical role of nonlinear modeling frameworks in portfolio risk assessment, hedging design, and systemic risk monitoring, particularly in emerging economies such as India, where global commodity and digital asset shocks increasingly impact domestic financial stability. Researchers may leverage this study in future investigations employing higher-dimensional copulas or time-varying fuzzy systems to capture evolving dependence structures within global financial networks.

This study concentrates on the Indian stock market, offering insights into regime-dependent co-movements among cryptocurrencies, crude oil, and equity markets within this emerging economy. However, the results may not be directly applicable to equity markets in other countries. Comparative studies on international equity markets are necessary to contextualize these findings more broadly. The analysis omits fixed-income instruments, such as bonds, which are crucial for diversification and portfolio management due to their sensitivity to interest rate dynamics affecting equities, oil, and digital assets. Incorporating fixed-income assets in future research would provide a more comprehensive understanding of cross-asset dependencies and hedging potential. Furthermore, the identified relationships reflect statistical dependencies rather than structural links, limiting their predictive utility for hedging. The study does not primarily address the temporal stability of these dependencies, which is vital for developing robust long-term diversification and hedging strategies. Future research should investigate the persistence and evolution of these relationships over time to enhance practical relevance.

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ABOUT THE AUTHOR / ИНФОРМАЦИЯ ОБ АВТОРЕ

Prabhat Mittal — PhD, Professor, Satyawati College, University of Delhi, Delhi, India
Прабхат Миттал — PhD, профессор, Колледж Сатьявати, Университет Дели, Дели, Индия
https://orcid.org/0000-0001-5352-7955
p.mittal@satyawatie.du.ac.in

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