

The Appraisal of Assets' Fair Value Using the Real Options Technique

Alexander Gulabyan

BA in Economics, International Finance Faculty, Department of World Economy and World Finance, Financial University. Scientific advisor: Tsvirko Svetlana Eduardovna, Associate Professor, PhD in Economics

Abstract

The goal of this paper is to analyse and systematise the possible approaches to real options valuation, especially when considering the practical aspects of their application in real-life valuation problems. Therefore, the paper sets the following tasks:

To outline the concept of fair value and analyse the traditional approaches to its calculation in the context of asset valuation

To define the real-option approach to fair value estimation and analyse its theoretical background

To determine the role of the real options approach in the traditional system of valuation techniques

To analyse the practical aspects of their application in valuation problems considering the corresponding examples

To provide the real-life example of this technique applied in current market conditions using the recent data.

The object of this research is the option pricing models, and the subject is their application in estimation of real options embedded in corporate valuations, particularly considering the side.

Keywords: real option valuation; asset valuation; fair value estimation; real options approach

JEL Classification: G11

© Alexander Gulabyan, 2020

Introduction

The relevance of the topic can be explained by the fact that asset valuation has always been one of the key pillars in finance playing a significant role in various economic relationships, is an essential element of any investment decision-making process and corporate valuations in general. Therefore, it is crucial for assets or investment projects to be fairly valued to prompt the right decisions and strategies to be adopted by the economic agents.

There are several traditional approaches to fair value valuation each of which has several methods with its relative advantages and drawbacks and thus may appear to be the best valuation tool given the corresponding conditions and the scope of available data. However, despite the variety of classical valuation techniques, there are still situations when their application is not effective or even useless.

In business practice, it often turns out that companies can react to changes in the market

situation, which makes it possible to adjust the incurred investment outlays. The additional value added to investment projects is, therefore, a premium received for the company's adjustment to changing market conditions.

In the traditional approaches, the value of such flexibility is neglected, because of which the evaluation of investment projects is often underestimated.

This value-adding flexibility is known as a real option that can be defined as a right (but not an obligation) to change a decision regarding an investment project when new information appears.

The *real-option approach* to valuation thus aims to capture the effect of these omitted value-drivers to assess the investment projects more fairly. Therefore, it may lead to the situation when the bad project, initially estimated by traditional methods, will appear to be profitable after accounting for the lurking investment opportunities measured by the real-option approach.

Although being widely-used in investment project appraisal the real-option approach may be also applied in the valuation of assets that do not generate any cash flows, but instead provide its holder with a right to earn income under a certain course of action — those generally include the intangibles such as licenses or exclusive patent rights to produce a particular product that will provide their holder with some cash flows in future; another good illustration is the unexplored reserves of natural resources. Valuation of such assets using the classical techniques will generally yield inaccurate results or maybe impossible at all, thus making the real-option technique even more essential in this case.

All these factors make the subject of the real options approach to valuation to be of practical worth in the field of corporate finance.

The real option valuation technique is not new. However, most papers on this topic consider the specifics of its application in business valuation and investment projects' appraisal. Typical examples include the research papers by Donald and DePamphilis, Mun J, Limitovskiy M.A., Pirogov N.K., Krukovskiy A.A., Huchzermeier A., Loch C., Bruslanova N., and others.

At the same time, there is a relatively small number of works concerning the aspects of real options approach application in the valuation of assets of special option-type such as patents and undeveloped reserves of natural resources. The methodology of real options approach application in the valuation of such assets was mainly developed by Aswath Damodaran in his books on valuation (2006, 2012) which I used as a key theoretical background of this paper.

The practical part of this research is based on the information mainly obtained from the Bloomberg Terminal, official company's financial statements and press releases.

This paper includes the following sections: introduction, three parts, and conclusions. In the introduction, I substantiated the relevance of the chosen topic, its theoretical and practical significance, defines the degree of its elaboration, and the scope of relevant literature and information base. It highlights this paper's goals and tasks, formulates the object and the subject of the undertaken research.

Part 1 outlines the notion of fair value and the theoretical basics of traditional valuation tech-

niques. Part 2 focuses on option pricing models, provides insights to the real options approach to valuation and analyses practical aspects of its application, considering the corresponding examples. Part 3 provides the real-life example of this real options valuation technique application in current market conditions using the recent data. The conclusion of this thesis summarises the key theoretical results of the undertaken research and provides practical recommendations on the application of the real options approach.

Theoretical Basics of Asset Valuation

Meaning of Asset Valuation and Concept of Fair Value

Asset valuation merely accounts to the value assigned to a particular property, such as stocks, options, bonds, buildings, machinery, or land, which is estimated usually when a firm or asset is to be disposed of, insured, or taken over.

There are many reasons for companies to undertake this procedure, including the following:

It allows to determine the right price for an asset, especially in cases of its acquisition or disposal and is beneficial for both parties of such a transaction, since neither the buyer nor the seller will pay more or receive less than the asset value

Every person or legal entity owning property or other assets has to pay taxes on them, hence asset valuation provides with an accurate estimation of the tax base for this purpose and so of the corresponding amounts of tax expenses incurred

It facilitates the business valuation process particularly in cases of merges between two or more companies or take-overs

Asset valuation procedure can also be undertaken for the lender needs when estimating the possible loan amount that can be paid back by the company offering its assets as collateral to be transferred to a lender in cases of the company's insolvency

Companies, especially public ones, are regulated, and hence are required, which means they need to present financial audits and reports for transparency. Part of the audit process involves verifying the value of assets.

According to IFRS13, the fair value is defined as the price that should be paid for an asset at its disposal or the price of transferring liability in the context of the simple transaction between independent of each other and knowledgeable

market participants acting in accordance with their financial interests at the measurement date under current market conditions, although in some cases observable market transactions or some other information might not be available for analysis. Thus, such a definition of fair value has the following key implications:

Fair value is the sale price of an asset — not the purchase one since sometimes these two prices are different (particularly for financial instruments)

Fair value is a market quotation. When measuring fair value, it is necessary to match it with the perceptions of market participants and thus exclude any uniqueness associated with the specific conditions in which the seller is placed

Market participants are independent, knowledgeable, willing to make a transaction and have access to the market

The intended sale transaction must be “normal” and not forced

Fair value should be tied to the measurement date.

Fair value measurement also implies that the transaction of asset disposal or liability transferring takes place in the principal market for the asset or liability or, in the absence of the former, in the most advantageous market for that asset or liability.

The principal market is the one with the greatest volume and level of activity for the asset or liability to be accessed.

The most advantageous market is the one, maximising the value to be received for the asset or paid to settle the liability after transportation and transaction costs. However, these two terms often coincide.

In spite of the assumption that the corresponding transaction is to be made in a principal or the most advantageous market conditions, the fair value itself is calculated before any adjustments for transaction costs that merely characterise only the transaction but not the involved asset or liability. However, in cases when location matters, the market price is adjusted for the expenses incurred to transfer the asset to that concrete marketplace.

All the data used by companies to measure fair value fall into three categories defined as corresponding levels comprising the hierarchy of fair value:

First level: observable data on identical valuation objects

Second level: observable data on similar valuation objects

Third level: unobservable data.

First level data include quoted prices in active markets for identical assets or liabilities that a company can receive at the measurement date. Such prices are the most reliable evidence of fair value and should be used without adjustments to estimate fair value whenever possible. This type of data is generally available in the currency, stock, brokerage (intermediary), dealer markets, and “from principal to principal” markets as well (where operations are carried out without intermediaries).

Second level incorporates the following data:

Quoted prices for similar (but not identical) assets or liabilities in an active market

Quoted prices for identical or similar assets or liabilities in markets that are not active.

Last level unobservable data is based on assessment and professional judgment. Such categorisation allows users of financial statements to objectively assess the quality of fair value estimates, since the higher the level of data used in measuring fair value, the higher is the quality of such estimates.

Thus, valuation methods used to measure fair value should maximise the use of observable data and minimise the use of unobservable data.

When measuring fair value based on unobservable data, a company can start with its own estimates, but must make adjustments if there is strong evidence that market participants will use different data or if the company has a piece of specific information used in its fair value estimations that is not available to other market participants. There is no need to spend a lot of effort to obtain information about the perceptions of market participants. However, one should consider any available information when making a fair value assessment.

Companies are obliged to provide detailed information on the fair value measurement process. Therefore, it is important to know which valuation methods and data have been used, as well as the basic information about the assessed assets or liabilities, considered significant.

Fair value aims to increase the degree of objectivity, transparency, and relevance of the information in the company's financial statements. Being a business valuation measure, it does not

make any sense to businesses in the context of its taxation. Its major advantage lies in its immediacy, which provides an updated valuation of assets and liabilities. Historical data, such as acquisition and production costs, does not provide its users with accurate and valid valuation estimates. Therefore, fair value assessment is known to be the best way to ensure the success of any investment and is also used as a basis for future cash flows prioritisation.

One of the advantages of fair value measurement from the point of its objectivity is the consideration of such factors as risks inherent to business activities. Since fair value is a market valuation, it is determined by the perceptions of market participants would use in relation to the value of an asset or a liability, including risk assumptions.

This remark is particularly relevant for the income approach to valuation when a fair estimate is calculated at the present value that will be discussed further in this chapter. In many cases, the amount and timing of cash flows are uncertain. Even the fixed amount stipulated by the contract, such as loan payments, is uncertain if there is a risk of default. For these market participants generally demand compensation (that is, risk premium) for accepting the risk inherent to cash flows associated with a particular asset or liability. Therefore, the fair value estimate should include a risk premium; otherwise, it will not represent the fair value.

Traditional Assets' Valuation Techniques

IFRS13 divides classical methods of asset valuation into 3 main groups, depending on the approach they are based on:

Methods of income approach, including discounting cash-flow (DCF) and direct capitalisation techniques

Market or sale comparison methods

Methods of valuing assets at their liquidation value or replacement cost, that constitute the cost approach.

The fair value hierarchy assigns priority to data rather than the corresponding valuation method; thus the fair value estimated on the basis of any of the methods may be attributed to each of its three levels, depending on which type of data is used.

Therefore, there is no universal valuation technique that will always yield the best result in any

case, since each of them might be more appropriate than others, depending on the particular circumstances, usually including the following factors to be considered by the investor or another party undertaking asset valuation when choosing the optimal technique:

The reasonably available information about the valuation object

The market conditions (for ex. the optimal valuation technique may vary, depending on whether the market is of a bullish or bearish type)

Investment horizon (for ex. some technique may perform better when measuring the fair value of long-term investments as compared to other methods and vice versa)

The life cycle of the investment object

The nature of business, where the examined asset is employed, as well as the type of the industry where this business is undertaken (for ex. some methods may perform better at capturing the volatile or in contrast cyclical nature of business).

Let us briefly discuss each approach in a bit more depth. The key notion of the *income approach* is that the asset fair value is estimated based on the present value of the expected future cash flows it will generate. However, each of its methods has some specific aspects which we must take into account.

Direct capitalisation model implies that asset will generate the same cash flows for each year of its holding or assumes that their growth rate is moderate and predictable. Hence the fair value (price) of the corresponding asset is found by capitalising its expected future cash flow for the one year or its average expected future cash flow to be received for the whole holding period (in case if the cash flows are not the same for each year) given the appropriate capitalisation rate which is the required rate of return on the asset being assessed that is generally equivalent to cost of equity or the WACC, depending on the capital structure of the business, where this asset is employed. (see formula 1.1 for direct capitalisation model¹)

$$P = \frac{CF}{r}, \quad (1.1)$$

where

CF — cash flow for one year or average cash flow

¹ The formula is a special case of general DCF model, given constant cash flows received forever, derived using formula of the sum of the infinite geometric progression.

r — capitalisation rate (required rate of return on asset).

However, the assumptions underlying the direct capitalisation technique seem to be rather unrealistic, making DCF model more flexible in this sense and yielding more accurate results, given the availability of all the necessary information, since it takes into account each individual expected cash flow that will be received in future and is assumed to arise evenly at the end of the each year during the whole asset holding period, including the expected cash flow from the asset's possible resale at the end of this period or the asset's scrap value (in cases if it is held till the end of its useful life). Hence the fair value of an asset under DCF model is defined as the sum of the present values of all the expected future cash flows received from the asset during its holding period, which formally is equivalent to discounting each future expected cash flow given the capitalisation (discounting) rate, which is generally assumed to stay constant during the asset holding period for simplicity, and finding their sum. (see formula 1.2)

$$P = \sum_{i=1}^n \frac{CF_i}{(1+r)^i} + \frac{R}{(1+r)^n}, \quad (1.2)$$

where

CF — cash flow for i -th year (n = last year of holding an asset)

R — cash flow from asset disposal at year n or asset's scrap value

r — capitalization (discounting) rate.

Since the DCF approach accounts for each individual potential future cash flow to be received from the asset also considering the time value of money, it is generally accepted as a primary asset valuation technique.

However, in real-world, any business usually operates under a certain degree of uncertainty. In this case, the timing of expected cash flows and their amount even for the first year in future usually cannot be forecasted for sure, as well as the level of appropriate capitalisation rate, making the methods of income approach absolutely useless in the assessment of asset fair value due to the lack of necessary information on the inputs required to estimate the corresponding amounts.

The other two asset valuation approaches — the market and the cost-based, are not sufficiently

influenced by business uncertainties since they don't generally require forecasting, but instead, use present data in fair value assessment, hence are preferable in this sense.

The market or sales comparison approach defines the asset fair value based on the price and other relevant information of market transactions involving similar or comparable assets. The market approach methods are mostly used in valuing unquoted equity instruments are generally related to the data sources used (for instance, quoted prices of public companies or prices from merger and acquisition transactions). Such relevant information used in fair value estimation under this approach usually includes the following:

Transaction price paid for an identical or a similar instrument of an investee

Comparable company valuation multiples derived from quoted prices (i.e. trading multiples) or from prices paid in transactions such as mergers and acquisitions.

Cost approach determines the asset's value based on the amount of expenses required to be incurred for its acquisition or production and, also incorporates several valuation methods, including the following:

Historical cost method

Replacement cost method

Replacement cost method.

The historical cost technique lies in identifying the actual costs incurred in the production of assets at prices effective on the day these costs have been incurred.

According to the other two methods, the asset fair value is estimated as the amount required to reproduce or replace it with a similar asset of the same production capacity. Hence the asset price under cost approach is equalised either to its reproduction or replacement cost, depending on the method applied. However, one should distinguish between these two terms.

Reproduction cost is the one required to reconstruct the analogous asset given the materials and technology available at the date of the assessment object creation.

Replacement cost, in contrast, is the amount required to build a similar asset of the same production capacity using the resources and technology available at the date of the asset assessment.

The asset fair value can also be measured based on its liquidation value. However, the asset value

based on this method does not correspond to the its liquidation value, that reflects the most likely price at which the asset may be alienated during its exposure time, which is less than the one under market conditions, given the seller is forced to make a transaction of this asset's disposal.

Thus, determination of the asset's liquidation value, as opposed to the market one, requires accounting for the effect of extraordinary circumstances, forcing the asset to be disposed under conditions that do not correspond to market ones.

Asset valuation plays a significant role in various economic relationships, being a crucial element of any investment decision-making process and corporate valuations in general.

Traditional asset valuation techniques include several methods concerning different aspects of this process, each of which has its own strengths and weaknesses.

However, given the availability and predictability of data on the corresponding variables such as future cash flows and discount rates the DCF approach is generally considered as the most accurate and superior asset valuation technique since it takes into account the potential income the asset is expected to generate each future period of its remaining useful life with regard to the time value of money.

Real Options Approach to Asset Valuation

Definition and Types of Real Options

An option is a right, but not the obligation, to buy, sell, or use an asset for a period in exchange for a specific fixed amount of money, defined as option strike or exercise price. Options providing its holder with the right to buy an asset are generally referred to as call options, while those granting the right to sell are known as put-options. Those traded on financial exchanges are called financial options.

Options that involve real assets, such as licenses, copyrights, trademarks, and patents, are referred to as real options. Other examples of real options include the right to buy land, commercial property, and equipment. Such assets can be valued as call options if their current value exceeds the difference between the asset's current value and some pre-set level. For example, if a business has an option to lease office space at a predeter-

mined price, the value of that option increases as lease rates for this type of office space increase. The asset can be valued as a put option if its value increases as the value of the underlying asset fall below a predetermined level. To illustrate, if a business has an option to sell an office building at a pre-set price, its value increases as the value of the office building declines.

The concept of real options was proposed in 1977 by Stewart Myers. Originally, the term "real option" meant the undefined benefits of the investment project (Myers, 1977, p. 150). It was not until the early 1990s that the concept of real options was used in practice to evaluate investment projects.

Real options valuation techniques are widely applied in investment projects characterised by a high level of risk and flexibility, allowing the decision-makers to actively respond to market changes during the project.

Real options reflect management's ability to adapt and later revise corporate investment decisions. They can impact substantially the value of an investment in a single project, which is generally underestimated when assessed using the standard DCF model since they account for the larking options that may be embedded in investment project and add up to its value and hence should be considered when appraising such investments. These options include actions that may be applied by the management in the course of realisation of the investment project to increase its value. However, as highlighted by Aswath Damodaran in his book on valuation (2006, p. 51), these actions must satisfy the following option recognition criteria to be qualified as real options:

An option should provide the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price at or before the expiration date of the option

There must be a clearly defined underlying asset whose value changes over time in unpredictable ways

The payoffs on this asset (real option) must be contingent on a specified event occurring within a finite period.

The similarity of real options to financial options results from the following factors:

The real option is the right to take a specific action

The option is exercised when it is beneficial to the buyer

They can be both call (call) and sale (put) options

The value of the option is the higher the uncertainty

The payoff function is asymmetric: potential losses are limited, and potential profits can be high.

Although being widely-applied in investment project appraisal the real-option approach may also be used in the valuation of assets that do not generate any cash flows, but instead provide its holder with a right to earn income under a certain course of action — those generally include the intangibles such as licenses or exclusive patent rights to produce a particular product that will yield some cash flows in future; another good illustration is the undeveloped reserves of natural resources. Valuation of such assets using the standard DCF approach will generally provide erratic estimates or maybe impossible at all, thus making the real-option technique highly valued in this case.

Despite the advantages of the extended flexibility in asset and investment project valuation, real options can be costly to obtain (e.g., the right to extend a lease or purchase a property), complex to value, and dependent on problematic assumptions — these are the main drawbacks of this approach. In this case, they should not be pursued unless the firm has the resources to exploit the option, and they add significantly to the value of the firm.

As also noted by Damodaran, there has to be a restriction on competition in the event of the contingency for an option to have significant economic value, since in a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value. He also mentioned another real options value driver — the degree of their possible exclusivity, depending on whether only their holder may take advantage of the contingency or somebody else also has such an opportunity, and if there is no exclusivity at all, then there is no option value as well. In this sense, real options become less valuable as the barriers to competition become less steep.

According to a recent survey (Horn et al., 2015, p.17), real options are used relatively infrequently by corporate chief financial officers (CFOs) but tend to be more common in the energy and bio-

tech industries. In these industries, investments tend to be large, long-lived, and subject to a wide range of outcomes.

Thus, the real options valuation technique is mainly used in the following areas:

- Research and development projects

- Projects in the mining industry

- Investment projects related to modern technologies (high-tech)

- Projects in universally understood human and intellectual capital.

However, in recent years, because of the growing interest in the subject of real options, the field of this method's application has extended and includes the following spheres:

- e-business

- venture capital projects

- start-up projects

- IT infrastructure

- e-commerce, m-commerce

- FMCG

- production industry.

There are several types of real options that generally may be embedded in the investment, depending on the possible course of actions adopted by the company management, such as:

- Option to delay (management may decide to delay or defer the investment project)

- Option to expand (the company may expand by entering new markets and developing new products at later stages of the investment project, based on the realised favourable outcomes at its early stages)

- Option to abandon (management may stop the production or abandon the investment project if the outcomes are unfavourable at its early stages).

Real Options Valuation Models

Option Pricing Models: Theory with Examples

The techniques of real options valuation are like those used in case of financial options value estimation and include the application of the generally known option-pricing models:

- The binomial model (usually accompanied with decision tree construction)

- The modified version of the Black-Scholes model.

As highlighted by Damodaran, both these methods incorporate the approach of a replicat-

ing portfolio and hence are mostly applicable for real option valuation when the following conditions are satisfied:

The underlying asset is traded — this particularly allows for the possibility of building replicating portfolios, except the observable prices and volatility being the model inputs

There is an active market for the option itself

The cost of exercising the option is known with some degree of certainty.

However, when using these models to value real assets, we must take the risk that the obtained value estimates may be biased as compared to the market price due to the difficulty of arbitrage. The key notion of this approach is to replicate the same cash flows generated by the option being valued, using a combination of risk-free borrowing/lending and the underlying asset. For instance, the call-option can be replicated by borrowing some amount of money at a risk-free borrowing rate and then buying some number of the underlying assets (e.g. shares). In contrast, put-options instead are replicated by the initial sale of some number of the underlying assets and then lending at a risk-free lending rate. The risk-free rate of borrowing is assumed to be equal to the one of lending, and the number of underlying assets bought or sold in this case is referred to as option delta (Δ).

The option value cannot be negative as in case of the worst outcome, it is not exercised so that its holder does not incur any losses. Thus, call and put option may yield the following payoffs as graphically represented in Figures 1 and 2 correspondingly:

Thus, given the corresponding risk-free rate (r) the European call's and put's price (denoted as c and p) with time to maturity of t years may take the following values:

$$\begin{aligned} \max[0; S - Xe^{-rt}] &\leq c \leq S \\ \max[0; Xe^{-rt} - S] &\leq p \leq Xe^{-rt} \end{aligned}$$

As also noted by Damodaran in his book (2006, p. 832.), since the time interval at which the option can be exercised (t) is shortened, the limiting distribution, as $t \rightarrow 0$, can take one of two forms:

If price changes become smaller as t tends to 0, the limiting distribution is the normal distribution and the price process is a continuous one

If price changes remain large as t tends to 0, the limiting distribution is the Poisson distribution, i.e., the one that allows for price jumps.

The Binomial Model of Option Pricing

The binomial model reflects the classical mechanism of option pricing and incorporates the idea of the replicating portfolio. This model provides with more accurate estimations in cases of several sources of uncertainty as compared to Black-Scholes model and usually includes the construction of decision trees with each of its nodes being the best estimate for the option value at the corresponding future time-period. There are some key assumptions underlying this model:

There only two possible scenarios at each future period — the best and the worst, so that the value of the option or the underlying asset may either increase or decrease as compared to the previous period

The underlying asset does not pay any dividends

No arbitrage is possible

The risk-free rate of borrowing/lending (r) is constant throughout the life of the option

Markets are frictionless, i.e. there are no taxes and no transaction cost

Investors are risk-neutral.

The mechanism of the option-pricing under binomial model could be illustrated by the following simple example of pricing a call-option on the stock using this approach.

Example 1. Suppose that some stock A today ($t = 0$) is priced at $S_0 = \$150$ and the next year ($t = 1$) the price is expected either to rise up to $S_u = \$170$ or drop till $S_d = \$130$.

Given the payoff structure on the call depicted in Figure If the exercise price of the call option on stock A is $X = \$160$, then its corresponding payoffs at $t=1$, given good or bad scenarios, can be written as:

$\text{Max}[S_u - X; 0] = \10 or $\text{Max}[S_d - X; 0] = \0 correspondingly. (see Figure 3)

The question is what the current price of the call-option is?

Solution. To replicate the call-option payoff we must borrow money (or sell a bond with face value of B) and buy N shares of stock A, so we have to find such N and B that will equalise our portfolio payoffs with the option payoffs given

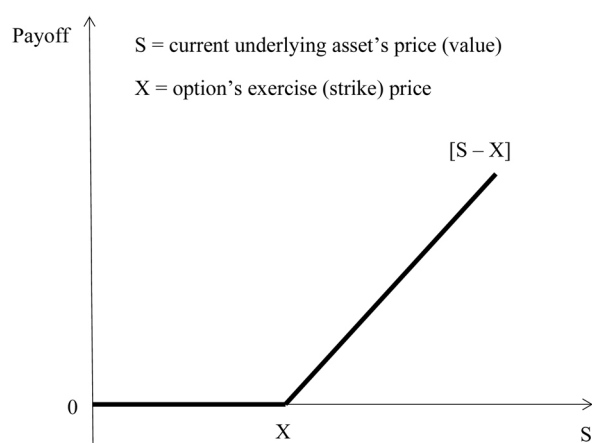


Figure 1. The payoff on call option

Source: The author.

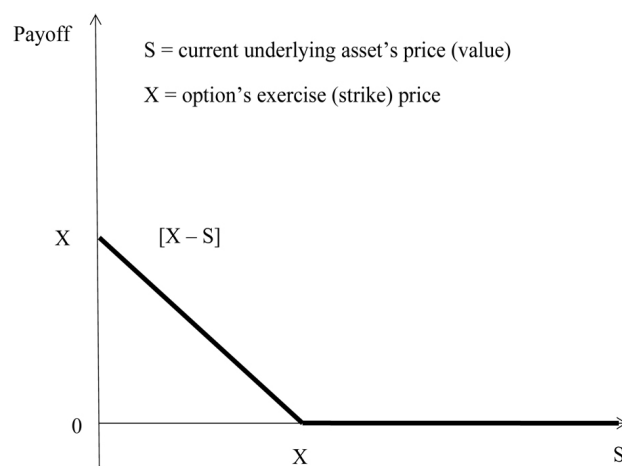


Figure 2. The payoff on a put option

Source: The author.

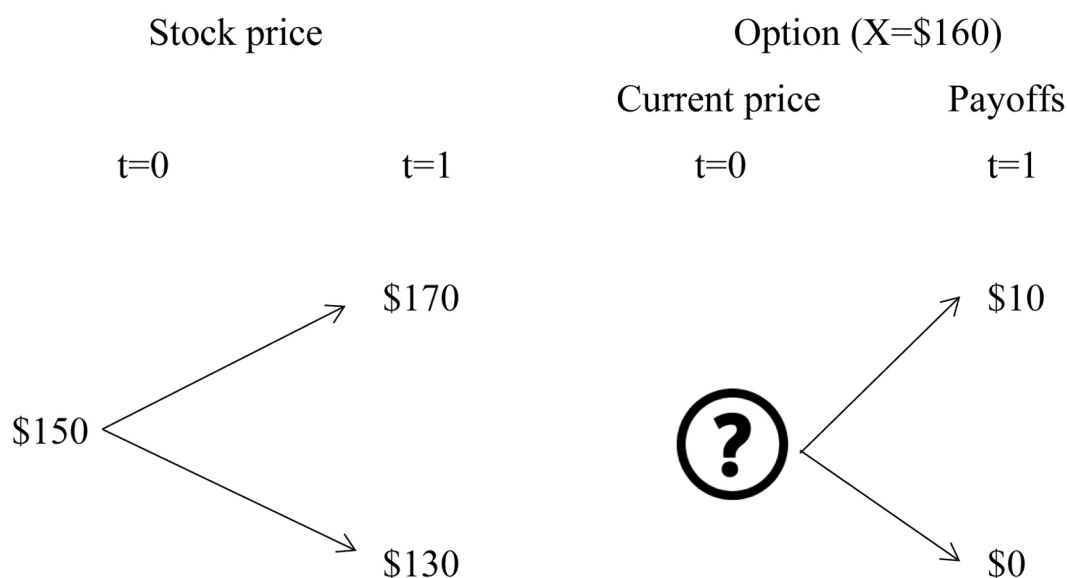


Figure 3. 1-step decision tree for a 1-period binomial option pricing model

Source: The author.

any scenario — good or bad. Formally, we should solve the following system of equation with respect to N and B:

$$\begin{cases} 10 = -B + N * S_u \\ 0 = -B + N * S_d \end{cases} \Rightarrow \begin{cases} 170 * N - B = 10 \\ 130 * N - B = 0 \end{cases} \Rightarrow \begin{cases} N^* = 0.25 \\ B^* = 32.5 \end{cases}$$

Since our stock/bond portfolio has the same payoffs as the option, the option and the portfolio must have the same value today, or else there will be the arbitrage opportunity that contradicts the

model assumptions. Hence the current value of the call-option equals to the today (discounted) value of our portfolio. Assuming risk-free rate (r) equal 5 per cent we have:

$$\frac{-B^*}{1+r} + N^* * S_0 = \frac{-32.5}{1.05} + 0.25 * 150 = 6.55.$$

So, the current value of the call-option equals to \$6.55. It was a simple example to illustrate how the replicating portfolio approach is embedded in the binomial model. In reality, the exact future stock (or another underlying asset) price is unknown. However, one can predict the level of its

increase (u) or decrease (d) at each time interval, using the following formulas:

$$u = e^{\sigma\sqrt{h}} \quad (2.1)$$

$$d = \frac{1}{u}$$

where

σ = standard deviation of stock price

h = time interval after which the price will change (as part of the year)

$u > 1, 0 < d < 1$.

If current stock price (S_0) will either increase by y_u per cent or fall by y_d per cent in each succeeding time interval, then the stock price in the next period will take one of these two possible values:

$S_u = S_0 * (1 + y_u\%) = S_0 * u$ — in case of stock price increase

$S_d = S_0 * (1 - y_d\%) = S_0 * d$ — in case of stock price drop.

E.g.:

if $u = 1.35$, then $y_u = 1.35 - 1 = 0.35 = 35\%$

if $d = 0.7$, then $y_d = 1 - 0.7 = 0.3 = 30\%$.

Given there are only 2 possible future scenarios of a stock price change and that we will anticipate the increase in stock price with some probability π , the stock price will fall with the probability of $(1 - \pi)$.

Assuming risk-neutrality one can calculate π by the following formula:

$$\pi = \frac{e^{-rh} - d}{u - d}, \quad (2.2)$$

where r = annual risk-free rate.

In cases of one-period models ($t = 1$) when the call/put option expires in one period time its price (c or p) is equalised with the discounted (present) expected value of its future payoffs in case of the current stock price (S_0) increase/fall up to S_u and S_d correspondingly (P_u and P_d).

$$c \text{ or } p = e^{-rh} (\pi P_u + (1 - \pi) P_d)$$

for call-option: for put-option: X = option exercise price

$$P_u = \text{Max} [S_u - X; 0] \quad P_u = \text{Max} [0; X - S_u]$$

$$P_d = \text{Max} [S_d - X; 0] \quad P_d = \text{Max} [0; X - S_d]$$

The similar approach is used when dealing with two or more period option pricing binomial models ($t > 1$). First one should define the possible option payoffs in the last period and then calculate

their expected value in the preceding period, using the same technique, and so on, moving back to the present period to determine the current price of the option.

Let us consider the following example to get a general idea. Assume a call option expiring in 6 months with a strike price (X) of \$15 and a current underlying stock price (S_0) is \$12. Risk-free rate (r) = 5 per cent. Every three months, the underlying stock price may either increase or drop by 30 per cent. In this case, we have:

$u = 1.3 \quad d = 0.7 \quad h = 0.25$ (for 3 months = $3/12$ (= $1/4$) of a year)

$$\pi = \frac{e^{-rh} - d}{u - d} \approx 0.48 \quad (1 - \pi) = 0.52$$

What is the current price of the call option (= c)? We'll construct a 2-step decision tree to deal with this problem, with each of its nodes indicating the underlying stock's price (S) in a particular period (t) in case of both possible scenarios: price going up or down (S_u and S_d) with the corresponding option payoffs (P_u and P_d) (see Figure 4).

Option payoffs at $t = 1$:

$$P_u = e^{-rh} (\pi P_{uu} + (1 - \pi) P_{ud}) \approx 0.99 * (0.48 * 5.28 + 0.52 * 0) \approx \$2.51$$

$$P_d = e^{-rh} (\pi P_{du} + (1 - \pi) P_{dd}) \approx 0.99 * (0.48 * 0 + 0.52 * 0) \approx \$0.$$

Now we can calculate the current price of our call-option:

$$c = e^{-rh} (\pi P_u + (1 - \pi) P_d) \approx 0.99 * (0.48 * 2.51 + 0.52 * 0) \approx \$1.19.$$

However, it was a relatively simple example. In reality, the underlying stock price may change each minute and the period of options expiration may be much longer, turning its price estimation into a rather tedious and complicated process.

It is probably the main drawback of the Binomial option pricing model that usually exceeds its relative advantage in the accuracy of the obtained estimation results in case of several sources of uncertainties as compared to Black-Scholes model, making the latter more effective in practice. Also, as highlighted by A.A. Kruckovskiy (2008, p. 129) in his paper on real options, with the increas-

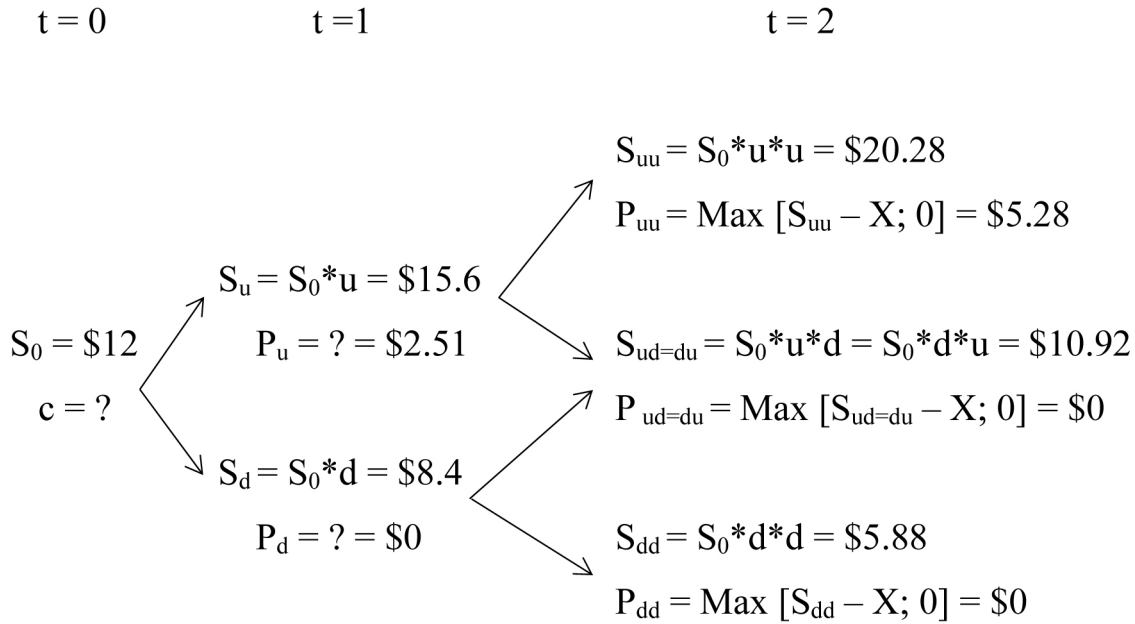


Figure 4. 2-step decision tree for a 2-period binomial option pricing model

Source: The author.

ing frequency of stock price change (i.e. with h tending to 0) both these models yield almost the same results.

The Black-Scholes Model of Option Pricing

The pricing analysis in the binomial model is based on the assumption that the underlying asset's prices are well-represented by a discrete time. However, in 1974, Fischer Black and Myron Scholes presented an option pricing model, allowing the time process for the underlying asset to be continuous. This analysis gave exact prices for puts and calls using a continuous time version of the replication strategy followed in the binomial methodology.

The Black-Scholes model included the same parameters as the Binomial one and was initially designed to value the options of the European type that may be exercised only at their expiration in contrast to Binomial model that may also be applied to the ones of the American type that may be exercised any time before their maturity date. The model assumes no sharp fluctuations in price and the returns on the underlying asset are expected to be normally distributed. Other underlying assumptions are like the ones of the Binomial model and include the following:

The underlying asset does not pay any dividends

No arbitrage is possible

The risk-free rate of borrowing/lending (r) and the volatility of the underlying asset price is known and constant throughout the life of the option

Markets are frictionless and efficient, i.e. there are no taxes or transaction costs, and the prices incorporate all the available information, so one cannot predict any of its possible movements.

Given these assumptions, the value of the call option (c) is calculated using the following formulas:

$$c = SN(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (2.3)$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

where

S — current underlying asset price

X — option strike (exercise) price

σ — volatility of the underlying asset price

t — life of the option (time till its expiration)

*values of $N(d_1)$ and $N(d_2)$ are taken from the cumulative distribution function (c.d.f.) of standard normal distribution

Despite being initially designed to value European call-options, the standard Black-Scholes model may be also applied to price an American calls that can be exercised at any time during their maturity period (t), since they were proved to have the same price as those of the European type, assuming that underlying asset does not pay any dividends.

From the formula (2.3) it is clear there are several factors influencing the price of a call-option that are summarised below:

The effect of the current underlying asset price (S): the Black-Scholes equation tells us that call option prices increase as the current spot asset price increases; This is pretty unsurprising as a higher underlying price implies that the option gives one a claim on a more valuable asset.

The effect of the exercise price (X): again, as you would expect, higher exercise prices imply lower option prices. The reason for this is clear: a higher exercise price implies lower payoffs from the option at all underlying prices at maturity.

The effect of volatility (σ): although outstanding outcomes (underlying price becoming very high) are rewarded highly, extremely bad outcomes are not penalised due to the kink in the option payoff function. This would imply that an increase in the likelihood of extreme outcomes should increase option prices, as large payoffs are increased in likelihood. The Black-Scholes formula verifies this intuition, as it shows that call prices increase with volatility, and increased volatility implies a more diverse spread of future underlying price outcomes.

The effect of time to maturity (t): call option prices increase with time to maturity for similar reasons that they grow with volatility. As the horizon over which the option is written increases, the relevant future underlying price distribution becomes more spread-out, implying increased option prices. Furthermore, as the time to maturity increases, the present value of the exercise that one must pay falls, reinforcing the first effect.

The effect of risk-free interest rates (r): when the risk-free rate rises — call option prices to grow. It is due to the same effect as above, in that the discounted value of the exercise price to be paid falls when rates rise.

Put-call parity

The Black-Scholes formula gives us a closed-form solution for the price of a European call

option under certain assumptions on the underlying asset price process. However, until now, we have said nothing about the pricing of put options. Fortunately, a simple arbitrage relationship involving put and call options allow us to do this. This relationship is known as put-call parity. In what follows, we assume the options have the same strike price (X), time to maturity (t) and are written on the same underlying asset.

Consider an investment consisting of a long position in the underlying asset with current value S and a put option with its price denoted as p , called portfolio A. The cost of this position is $S + p$.

Another portfolio denoted as B, comprises a long position in a call-option with its price denoted as c and lending Xe^{-rt} . Hence the cost of this position is $c + Xe^{-rt}$.

What are the possible payoffs of these positions at maturity?

Given the payoff structure on the put-option, depicted in Figure 2, the payoff on portfolio A can be written as follows:

$$\max [X - S; 0] + S = \max[X; S]$$

Similarly, the payoff on portfolio B can be written as:

$$\max[0; S - X] + X = \max[X; S]$$

Comparison of the above two equations implies that the two portfolios always pay identical amounts. Hence, using no-arbitrage arguments, portfolios A and B must cost the same amount. Equating their costs, we obtain:

$$S + p = c + Xe^{-rt} \quad (2.4)$$

Equation 2.4 is the put-call parity relationship. Given the price of a call (c), the current value of the underlying asset (S) and knowledge of the riskless rate (r), we can deduce the price of a put (p) using the following expression:

$$p = c + Xe^{-rt} - S$$

Substituting for c from the Black-Scholes formula (2.3) and rearranging the terms we get the formula for pricing a put-option:

$$p = Xe^{-rt} (1 - N(d_2)) - S(1 - N(d_1)) \quad (2.5)$$

Similarly, given the put price, we can deduce the price of a call with similar features.

The equation allows to deduce the effect of the Black-Scholes model's parameters on put prices:

The effect of the underlying price (S): for the opposite reason to that given for the call, put prices drop as underlying prices increase

The effect of the exercise price (X): similarly, put prices rise as exercise prices rise

The effect of volatility (σ): put options and call options are affected in identical ways by volatility. Hence, as volatility increases, put prices rise

The effects of time to maturity (t) — increased time to maturity will lead to a greater dispersion in underlying prices at maturity, and hence put prices should be pushed higher. However, as the holder of a put receives the exercise price, discounting at higher rates makes puts less valuable. The combined effect is ambiguous

The effect of the risk-free rate (r): puts are less valuable as interest rates rise, due to a higher degree of discounting of the cash received.

The Black-Scholes approach also incorporates the concept of replicating portfolio in its framework: to replicate the call option one should buy $N(d_1)$ of the underlying asset (i.e. $N(d_1)$ is the option delta(Δ)) and borrow $Xe^{-rt}N(d_2)$.

It'll be reasonable to add that this model can be adjusted to take dividends into account by adding dividend yield (y) as its new component and assuming it constantly throughout the options life, so the modified version of the Black-Scholes model takes the following form:

$$c = Se^{-yt}N(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (2.6)$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

The price of the put-option (p) may be also derived from the put-call parity:

$$p = Xe^{-rt}(1 - N(d_2)) - Se^{-yt}(1 - N(d_1)) \quad (2.7)$$

Moving back to the case of pricing the American call options that can be exercised any time before the settlement date it should be noted that call-estimates yielded by a modified version of Black-Scholes model are rather close to the price

of American call on the asset that pays dividends since. Generally, there are no clear benefits, and thus no solid reasons for early option exercise and so the ability to do so does not make any sense.

Early exercise is generally prompted by weird mispricing resulted from the technical or market-based reason causing a mess in the theoretical option's prices.

However, there are certain circumstances under which the early exercise will add up some value, thus resulting in a higher American calls' prices as compared to the European ones. Such circumstances imply the following favourable conditions to be satisfied:

The option is deep-in-the-money and has Δ (i.e. $N(d_1)$) equal or close to 1

The option has little time value

The dividend payment is relatively high, and its ex-date precedes the option expiration period (t).

Despite the limitations considering its underlying unrealistic assumptions particularly the Black-Scholes model is generally known to be the most optimal approach to option valuation mostly due to its relative simplicity and generality enabling to undertake rather complicated valuations without turning them into a time-consuming, exhausting process, just like in case of binomial decision tree construction, since all you need is to obtain the data on the model inputs and then substitute it into the corresponding formula to obtain the immediate result.

For this reason, the modified version of the Black-Scholes model is also frequently applied in real options valuation and hence will be used in our analysis.

Application of Black and Scholes Model in Real Option Valuation

Real Options on the Asset Side

Valuing product patent as a real option to delay

Application of Black and Scholes model in real options valuation we based on the book by Damodaran (2012). A firm with exclusive rights to a project or product for a specific period may decide to postpone the investment in a project or product development which is also known as exercising of the option to delay.

A product patent provides a firm with an exclusive right to develop and market a specific product and hence can be defined as a real call option to delay.

The underlying asset, in this case, is the project or product on which development a firm has a patent, and so the present value (PV) of the expected cash flows from developing the product today may be treated as its current value (S) while the option's exercise price (X) is equivalent to the cost of the product development. The volatility of cash flows or revenues received from current products can be used as a proxy to estimate the volatility of the underlying asset value (σ).

The patent expires when the exclusive rights to the product end. When the patent rights expire, excess cash flows associated with the holding of these rights vanish as other competitors will also be empowered to manufacture this product and reap the possible gains.

As highlighted by Donald DePamphilis in his book (2017, p. 299) on mergers and acquisitions, the opportunity cost of product development delay is equivalent to an adjustment of the initial Black-Scholes model that made it possible to expand its application in situations considering underlying assets with regular dividend payments. The payment of a dividend is referred to as a reduction in the stock value since such funds are not reinvested in the company to provide future growth. Hence, if the projected cash flows from the product arise evenly throughout the patent life (t) then with each year of the product development delay, the firm will sacrifice the potential cash flow for this year that could have been received. So, the annual cost of delay(y) in this case

may be calculated as $\frac{1}{t}$.

If cash flows are not anticipated to rise evenly, the cost of delay may be approximated as the ratio of the expected cash flow for the next period to the current value of the underlying asset(S).

The firm will exercise its patent rights only if the present value of the potential cash flows from the product development exceeds the present value of the associated costs. (i.e. if $S > X$) or else it can set the patent aside, thus escaping any associated future costs. Thus, the patent-holding may yield the following payoffs:

$$P_1 = S - X \text{ if } S > X$$

$$P_2 = 0 \text{ if } S \leq X$$

Given all these remarks, one can apply the Modified Black-Scholes option pricing model to value the product patent (see formula 2.6).

Let us consider the following example. Suppose a company A holds a patent on product B for the next 25 years (i.e. $t = 25$) and plans to manufacture and realise it by itself. PV of cash flows from introducing product B now (S) = \$5m and cash flows are assumed to arise evenly throughout the patent life; PV of development costs (X) = \$3m. Risk-free rate (r) = 6 per cent and the volatility in S (σ) = 0.5. What is the value of the patent? Finding the value of the patent is equivalent to the valuation of a real call option to delay.

$$\text{The annual cost of delay (y)} = \frac{1}{t} = \frac{1}{25} = 0.04$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = \frac{\ln\left(\frac{\$5m}{\$3m}\right) + \left(0.06 - 0.04 + \frac{0.5^2}{2}\right)25}{0.5\sqrt{25}} \approx 1.65$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx 1.65 - 0.5\sqrt{25} \approx -0.85$$

$$N(d_1) = \Delta = \text{Probability (Pr) } (Z \leq d_1), Z \sim N(0;1) = \text{Pr}(Z \leq 1.65) \approx 0.9505$$

$$N(d_2) = \text{Pr}(Z \leq d_2) = \text{Pr}(Z \leq -0.85) = 1 - \text{Pr}(Z \leq 0.85) \approx 1 - 0.8023 \approx 0.1977.$$

Having obtained all the necessary inputs, we can apply the Modified Black-Scholes model to get the answer:

$$c = Se^{-yt} N(d_1) - Xe^{-rt} N(d_2) \approx \$1.62m$$

Hence the patent on product B is approximately worth \$1.62m.

It should be noted that NPV of product B if introduced now equals $S - X = \$2m > c = \$1.62m$ (estimated value of a patent on product B), thus for company A it is better to introduce B now than wait till later times holding patent rights on it unexercised.

The real-option approach to patent valuation is best suited for firms of the following types:

New firms or start-ups having only one or two promising products in the corresponding niche, and at the same time earning little or no revenue or cash flow

Firms where product patents comprise the substantial part of their value and thus cannot be assessed using the traditional DCF technique.

Table 1
Black-Scholes model inputs for resources' undeveloped reserves valuation

Notation	Model input characteristics	Estimation hints and possible proxies
S	the current value of underlying asset = PV of estimated reserves of natural resources	sum of the projected future cash flows from the same or similar resources' exploitation discounted back at the development lag length. *could be approximated using the past average data on growth rates of cash flows from the analogous resources' reserves if the general trend is expected to remain relatively stable
X	PV of development costs	past data on the development costs of the same or similar resources
t	option life = expiration period of firm's right to exploit the resources' reserves	
r	risk-free rate	Treasury bond rate corresponding to t
y	the annual cost of delay	could be approximated as the ratio of average annual cash flow (or net sales) from the reserve to its current value (S)
σ	volatility in S	could be approximated by volatility in the same or similar resource prices

Source: The author.

The described model may also be extended to estimate the value of the firm, particularly of these types that could be obtained by summing its following three components:

PV of net cash flows from the firm's commercial products, discounted at the firm's corresponding WACC (estimated with DCF model)

Value of the firm's existing patents, obtained via application of the real-option approach

The expected value of commercial products that the company plans to generate in future from new patents that may be obtained because of its research and development (R&D) activities. (i.e. Value of new patents that will be obtained in future — R&D cost to be incurred to obtain these patents).

As argued by Damodaran (2006, p. 790), the value of the third element will depend on the expectations of a firm's R&D capabilities. Thus, given a firm earning its cost of capital from R&D, this component will turn to zero.

Valuing natural resources as real options

The same technique may also be applied in the valuation of the undeveloped reserves of natural resources. However, the notion of the modified Black-Scholes model inputs will be slightly different in this case (see table 2.1 for details).

In this case, the examined resource takes the role of the underlying asset which value is determined by two factors — the resource amount available for development and the price of the resource. A firm owning some undeveloped reserves is consequently provided with a right either to develop them and yield cash flows from generated resources (i.e. the value of the underlying asset (S)) or not if costs are equal or exceed the potential benefits. If we define resource development costs as X then the undeveloped reserves of resources may be presented as a real call-option with the following payoffs:

$$P_1 = S - X \text{ if } S > X$$

$$P_2 = 0 \text{ if } S \leq X$$

Once developed, the reserve may provide its owner with resources for sale only after some time which is defined as development lag. The cost of such delay is the lost potential cash flows that could have been received during the lag period. Hence the value of the reserves is discounted back at the development lag length using an annual cost of delay as a discounting rate.

Let us consider the following example. Suppose company A owns a natural gas reserve of 30m m³, and the PV of its development costs is estimated at \$10 per m³ of gas with the development lag (dlag) of 3 years. Assume each m³ of gas sold currently

yields a marginal profit of \$10 on average. The firm's rights to exploit the reserve will expire in 25 years. (i.e. $t = 25$). Once developed, the reserve is expected to provide about 6 per cent of its current value each year from sales of the generated resources. (i.e. $y = 0.06$). The volatility in prices for gas (σ) is 0.2, and the risk-free rate (r) is 8 per cent. What is the current value of the undeveloped reserves of gas?

Firstly, we calculate the current value of estimated gas reserves (S) and PV of total development costs (X):

$$S = \frac{\$10 * 30m}{(1 + y)^{dlag}} = \frac{\$300m}{1.06^3} \approx \$251.88m$$

$$X = \$10 * 30m = \$300m$$

Now we can estimate d_1 and d_2 :

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 0.82$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx -0.17$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.7954$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.4306$$

Having obtained all the necessary inputs, we can apply the Modified Black-Scholes model to get the answer.

$$c = Se^{-yt} N(d_1) - Xe^{-rt} N(d_2) \approx \$27.22m$$

Hence the undeveloped natural gas reserves are approximately worth \$27.22m.

The real-option approach to undeveloped resources reserves valuation is best suited for firms of the following types:

Firms where rights on natural resources reserves comprise the substantial part of their value

Firms which natural resources reserves are rather homogeneous so that the Black-Scholes model's corresponding inputs may be estimated with adequate accuracy.

The described model may also be extended to estimate the value of the firm, particularly of these types that could be obtained by summing its following two components:

PV of net cash flows from developed reserves of natural resources, discounted at the firm's corresponding WACC (estimated with DCF model)

We obtained the value of existing undeveloped firm's reserves via application of the real-option approach.

Thus, moving back to our example and, if PV of company's A net cash flows from the already developed reserves equals \$70m, its firm's value may be calculated using the following formula:

$$\text{Value of firm A} = \$70m + c \approx$$

$$\approx \$70m + \$27.22m \approx \$97.22m$$

In the event when firm's every single undeveloped reserve cannot be valued by real-option technique due to lack of data on the model's corresponding inputs the firm itself can be estimated as one call-option on its assets. However, as highlighted by Damodaran (2006, p. 797), the firm's value, in this case, is likely to be underestimated since the option on a portfolio on assets (i.e. a firm estimated as one call-option) generally worthies less than a portfolio of call-options on each firm's single asset (each firm's undeveloped reserves' resources).

Valuing Other Options Embedded in Investment Projects

Option to engage in other projects (expand)

As noted above the real options approach is also widely applied in investment project appraisal since it allows us to consider the possible options embedded in the project, that are neglected in the classical approaches. These lurking options add-up to the project's value, so there could be the case that a project initially assessed with traditional techniques yields negative NPV and hence is considered as bad one that shouldn't be worth taking but in fact, the true NPV adjusted to the effects of the corresponding options may be positive turning the bad investment into a good one increasing the investor's wealth.

One of the options adding-up to the project's value is the option to expand, since engaging in a project today may enable a firm to undertake other additional projects, so the resulting total NPV may be improved as compared to one of the initial projects.

Let us consider the following example. Suppose company A plans to introduce new product

Table 2
Black-Scholes model inputs for expansion option valuation

Notation	Model input characteristics	Estimation hints and possible proxies
S	the current value of underlying asset = PV of expected cash flows from expansion if undertaken now	*could be approximated using the past average data on growth rates of cash flows from the analogous products being introduced to the regional market (or the one of the relatively same size) if the general trend is expected to remain relatively stable
X	PV of expansion costs (additional investment)	past data on the expansion costs of the same or similar projects
t	the expiration period of expansion possibility	
r	risk-free rate	Treasury bond rate corresponding to t
y	the annual cost of expansion delay	assuming that cash flows will arise evenly throughout the expansion period this cost may be calculated as $\frac{1}{t}$
σ	volatility in S	could be approximated by volatility in the same or similar products' prices or using the volatility in the free cash flows or an enterprise value of the firm, its analogues or the whole industry on average

Source: The author.

B. A believes that its competitors won't be able to copy the product B due to the patent protection for at least 7 years, so A will enjoy a monopolistic position in B production and expects to receive cash flows from this product's realisation during the patent life period (i.e. $t = 7$ years).

Company A may introduce its product B only for local (city) market by spending \$300m. It has been estimated that the PV of potential cash flows from the local introduction will amount only \$200m and hence the project will yield negative NPV (NPV_B^1) of \$100m and thus should be rejected.

However, if the local introduction goes well the company A will be provided with an option to enter a regional market with its new product B by investing additional \$700m anytime during the next 7 years (until the expiration of the patent rights on product B) that is expected to generate the total PV of \$1,050m from the cash flows received.

The risk-free rate (r) is assumed to be 8 per cent, and the volatility in the similar product's prices (σ) is estimated as 0.3. This expansion option will add some value to the project that can be estimated using the modified version of the Black-Scholes model with the following inputs, as presented in Table 2.

From the information given we have:

$S = \$1050m$

$X = \$700m$.

Now we can estimate d_1 and d_2 :

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 0.35$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx -0.44$$

$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.6381$

$N(d_2) = \Pr(Z \leq d_2) \approx 0.3298$.

Finally, we can obtain the following call-option value:

$$c = Se^{-yt}N(d_1) - Xe^{-rt}N(d_2) \approx \$114.59m.$$

Hence the expansion option adds-up approximately \$114.59m to the projects value making its total NPV (NPV_B^T) positive:

$$\begin{aligned} (NPV_B^T) &= (NPV_B^1) + c \approx \\ &\approx \$-100m + \$114.59m \approx \$14.59m > 0. \end{aligned}$$

Table 3
Black-Scholes model inputs for project abandonment option valuation

Notation	Model input characteristics	Estimation hints and possible proxies
S	the current value of underlying asset = PV of expected cash flows from the project	*could be approximated using the past average data on growth rates of cash flows from the analogous projects being if the general trend is expected to remain relatively stable
X	the residual value of the project abandonment (value of Alpha's share at its disposal to Beta)	past data on the same or similar projects
t	the expiration period of the project abandonment possibility	
r	risk-free rate	Treasury bond rate corresponding to t
y	the annual cost of abandonment delay	assuming, that cash flows will arise evenly throughout the project life this cost may be calculated as $\frac{1}{n}$
σ	volatility in S	could be approximated by volatility in the cash flow from similar projects

Source: The author.

Given the expansion option, the project increases the company's wealth by approximately \$14.59m, and hence it should be definitely undertaken.

Option to abandon a project

The value of continuing a project given the remaining n years of its life should be compared to the value at its liquidation or disposal. If the former exceeds the latter, then the project should be continued – otherwise, it is better to be abandoned to save on its costs. It is the key notion of firm's another possible option that may be embedded in a concrete project and affect the decision on making the corresponding project's investment. A company can choose to shut down a project if its generated cash flows are far away from the expected amounts in order not to incur further losses which may consequently add value to the project.

Let us consider the following situation to get the idea. Suppose company Alpha considers undertaking a co-project with company Beta to produce product A. The project will last 20 years (i.e. n = 10). Alpha shares 40 per cent of the project's costs and potential gains and hence is required to invest \$300m with the PV of anticipated cash flows estimated as \$270m which yields a negative project's NPV of \$30m (NPV_A^1) thus turning the project in a total loss for Alpha.

However, Alpha is also given an option to sell its share of the investment to Beta anytime during the next 7 years for \$200m if it decides to exit the project.

This option adds some value to the initially unprofitable project that is equivalent to the price of the corresponding put-option and hence can be estimated using the modified Black-Scholes approach. The model inputs for this case are shown in Table 3.

The Alpha's cash flows from the project are assumed to arise evenly throughout its life, and the volatility in their PV-s (σ) was estimated as 0.4. The corresponding 7-year risk-free rate (r) is 7 per cent.

Obtaining the inputs for the model:

S = \$270m

X = \$200m

$$y = \frac{1}{n} = 0.05$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 0.94$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx -0.11$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.8277$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.4549.$$

Table 4
Black-Scholes model inputs for equity valuation

Notation	Model input characteristics
S	the current value of underlying asset = current firm's value
X	the nominal value of outstanding financial claims (zero-coupon bonds)
t	life of the option = maturity period of zero-coupon debt
r	risk-free rate = Treasury bond rate corresponding to t
σ	volatility in S

Source: The author.

Finally, we can obtain the corresponding put-option value:

$$p = Xe^{-rt}(1 - N(d_2)) - Se^{-yt}(1 - N(d_1)) \approx \$34m.$$

Hence the abandonment option adds-up approximately \$34m to the projects value making

its total NPV (NPV_A^T) positive:

$$(NPV_A^T) = (NPV_A^1) + p \approx \\ \approx \$-30m + \$34m \approx \$4m > 0.$$

Given the abandonment option, the project increases the Alpha's wealth by approximately \$4m and hence should be undertaken.

Valuing equity at firm liquidation as a real option

Equity may be referred to as a residual claim on the firm's value left after all other financial claims have been satisfied.

Hence in case of firm liquidation, equity-holders receive the entire firm's residual value left after discharging of debts and settlement of other liabilities. At the same time, the principle of corporate veil protects the equity holders by limiting their liability to the amount of their equity investments in case of the company's total financial claims' amount exceeding the firm value, so they risk to lose no more than the total nominal value of shares they hold.

Thus equity may be presented as a call option on the current value of the firm (i.e. the PV of firm's future free cash flows) being in the role of the underlying asset in this case (i.e. S) that

may be exercised only at firm's liquidation after redemption of the nominal value of its outstanding debt being the option's exercise price (X). This option thus may yield the following payoffs:

$$P_1 = S - X \text{ if } S > X \\ P_2 = 0 \text{ if } S \leq X$$

Assuming a firm has only outstanding zero-coupon bonds with a fixed maturity date in its debt structure and can be liquidated any time before this date then the option's life will coincide with the bonds' time to maturity and thus the value of firm's equity may be calculated using the classical Black-Scholes model with the following parameters as depicted in Table 4.

Let us consider the following example. Suppose, Company A is currently worth \$150m with the volatility in its value (σ) estimated as 0.3 and has only outstanding zero-coupon bonds on its debit account with a nominal value of \$130m and maturing in 12 years. Assuming the corresponding risk-free rate (r) equal to 12 per cent – what is the value of the company's equity and debt outstanding? From the information given we have:

$$S = \$150m$$

$$X = \$130m$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 2.04$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx 1$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.9795$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.8422.$$

Substituting the obtained results in the Black-Scholes formula we get:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \approx \$120.979m.$$

Hence the value of the company's A equity approximately equals to \$120.979m and thus the value of debt outstanding amounts to \$29.021m (i.e. $S - c$).

Now we assume that part of the company's assets was destroyed by a natural disaster so that its value plummeted to \$100m ceteris paribus. In this case, given the debt face value of \$130m exceeding the new firm's value, the

Table 5

Firm's A debt structure

Debt type (i)	Debt maturity	Face Value (FVi) (including expected interest/coupon payments)	Duration (Di) (in years)
1	Short-term	\$10m	0.3
2	10 years	\$50m	7
3	20 years	\$70m	15
4	Long-term	\$100m	19
$\sum FV_i$		\$230m	

Source: The author.

company is considered a troubled firm since it is likely to go bankrupt due to insolvency. What will be the new value of A's equity? Since all other factors except the firm's value stayed unaltered, our model parameters will take the following values:

$$S = \$100m$$

$$X = \$130m$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 1.65$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx 0.61$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.9508$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.7302.$$

Hence the new value of equity will be:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \approx \$72.589m.$$

The result indicates the fact that stock of troubled firms which is generally treated as worthless when accessed by traditional DCF technique, since the discounted nominal amount of firm's financial obligations exceeds its current value, still has some value due to the time premium on the option (i.e. firm's equity), suggesting that the underlying asset's value (i.e. the firm's value) may exceed the option's strike price (i.e. the face value (FV) of firm's debt) at some point in time until the end of the option's life, thus making real-option approach particularly worth applying in equity valuation

of essentially bankrupt firms. Although the model assumes that a firm has only one debt-issue of a zero-coupon bond, it could also be adjusted to value equity in firms with relatively complex debt structure by converting them into one equivalent zero-coupon bond.

The best way to do it is to estimate a face-value-weighted average of the durations corresponding to each of debt types and use it as a maturity period of zero-coupon debt (i.e. a life of the option considered (t)) in the Black-Scholes model.

Let us consider the following example. Suppose firm A has a complex debt-structure that is described in the table below.

What will be the corresponding face value of the zero-coupon bond equivalent to firm's aggregate debt (i.e. X parameter in the Black-Scholes option pricing model) and what will be this bond's maturity period (t)? The first part of the question requires to sum the interest and coupon adjusted face values corresponding to each of debt types (FV_i) to get the answer. Thus, the face value of the simulated equivalent zero-coupon bond (X) will be equal to $\sum FV_i = \$230m$.

As noted above, this bond's maturity period (t) may be approximated by a face-value-weighted average of these bonds' durations given the following formula:

$$t \approx \frac{\sum_{i=1}^n FV_i D_i}{\sum_{i=1}^n FV_i}, \quad (2.8)$$

where n = number of debt types in the firm's debt structure.

Hence, in this case, we have:

$$t \approx \frac{\sum_{i=1}^n FV_i D_i}{\sum_{i=1}^n FV_i} \approx \frac{\$10m * 0.3 + \$50m * 7 + \$70m * 15 + \$100m * 19}{\$230m},$$

where $t \approx 14.36$ years.

At the same time, we can note that the Black-Scholes model may overestimate the equity value when applied for companies financed by various debt instruments with different coupon/interest payments and terms to maturity (e.g. like firm A in the above given an example).

Since the option pricing model allows for only one input for the time to expiration, we must convert these multiple bonds issues and coupon payments into one equivalent zero-coupon bond. However, it is reasonable to add that results on firm's equity values provided by real options valuation technique will be more precise in cases when the variance in the firm's value may be estimated with adequate accuracy since the model is rather sensitive to this parameter.

When considering equity to be an option one may provide insights to the potential reason underlying the conflicts of interest between equity holders and bondholders. As suggested by Damodaran (2012, p. 1169) in his book on valuation since equity being treated as an option increases in value with volatility in firms value, then it may be the case that stockholders may prompt the company to invest in risky negative NPV projects to increase their wealth at the expense of the bondholders.

Let us consider the following situation. Suppose company A (from the previous example) is in the state before the disaster. Hence the firm's current value (S) is \$150m with the following capital structure, as calculated before:

$$\left. \begin{array}{l} \text{Value of equity: } \$120.979m \\ \text{Value of outstanding debt: } \$29.021m \end{array} \right\} S = \$150m$$

Now imagine that shareholders invest in the project with negative NPV of \$-10m that is rather risky thrusting the volatility in A's value (σ) to 0.6. What will be the firm's new capital structure? Using the initial information on the task from the previous example we have:

$S = \$150m - \$10m = \$140m$ (since the initial firm's value will be reduced by the project's negative NPV amount)

X = nominal value of zero-coupon debt outstanding = \$130m

t = time to maturity of zero-coupon bonds = 12 years

r = 0.12

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 1.77$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx -0.31$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.9614$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.378.$$

Thus, the post-project value of A's equity will be:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \approx \$122.96.$$

And the value of the firm's outstanding debt after its unprofitable investment will amount to \$17.04m (i.e. $S - c$). Hence the firm's resulting capital structure will be (initial amounts are placed in brackets for ease of comparison):

$$\left. \begin{array}{l} \text{Value of equity:} \\ \$122.96m \\ (\$120.979m) \\ \text{Value of outstanding debt:} \\ \$17.04m \\ (\$29.021m) \end{array} \right\} S = \$140m (\$150m)$$

The obtained results indicate that the company's equity value has improved by approximately \$2m at the expense of bondholders' wealth that has dropped by a substantial amount of nearly \$12m.

As also argued by Damodaran such conflict of company's primary stakeholders' interests given the option nature of equity may also be illustrated when concerning the situation of conglomerate mergers of firms with volatilities in their free cash flows (and thus in their value) being negatively or at least not highly positively correlated that generally is the case when merging firms operate in different economic sectors. As a result of such merger given the relatively small correlation between merging

firms the resulting volatility in a merged firm will be reduced as compared to an initial state before the merger — this could be derived from the variance property. Hence the equity holders will realize the substantial post-merger drop in their wealth while the bondholders will, in contrast, be better off. However, the adverse effect on shareholders' wealth may be partly offset by an additional bond issue.

Let us consider the following example. Suppose company A and company B operating in different industries and thus having the correlation coefficient between their free cash flows (ρ_{AB}) estimated as -0.3 (i.e. firms' values are negatively correlated) decide to merge.

The information on A and B is given below.

	A	B
Value of the firm (S)	\$200m	\$250m
FV of debt (zero-coupon bonds) (X)	\$130m	\$70m
Debt maturity period (t)	12 years	12 years
Volatility in S (σ)	0.3	0.4

The corresponding risk-free rate (r) is 12 per cent. What will be the value of equity and outstanding debt in the merged firm? First, let us calculate the equity and outstanding debt value of each firm before the merger. From the information given we have:

$$\begin{aligned} S_A &= \$200m \\ X_A &= \$130m \end{aligned}$$

$$d_1^A = \frac{\ln\left(\frac{S_A}{X_A}\right) + \left(r + \frac{\sigma_A^2}{2}\right)t}{\sigma_A \sqrt{t}} \approx 2.32$$

$$d_2^A = d_1^A - \sigma_A \sqrt{t} \approx 1.28$$

$$N(d_1^A) = \Delta = \Pr(Z \leq d_1^A), Z \sim N(0;1) \approx 0.9898$$

$$N(d_2^A) = \Pr(Z \leq d_2^A) \approx 0.8998.$$

Substituting the obtained results in the Black-Scholes formula we get:

$$c_A = S_A N(d_1^A) - X_A e^{-rt} N(d_2^A) \approx \$170.249m.$$

Hence company A initially has about \$170.249m of equity and thus the value of its debt outstand-

ing amounts to \$29.751m (i.e. $S_A - c_A$). The same steps will be taken to estimate equity and outstanding debt levels of company B:

$$\begin{aligned} S_B &= \$250m \\ X_B &= \$70m \end{aligned}$$

$$d_1^B = \frac{\ln\left(\frac{S_B}{X_B}\right) + \left(r + \frac{\sigma_B^2}{2}\right)t}{\sigma_B \sqrt{t}} \approx 2.65$$

$$d_2^B = d_1^B - \sigma_B \sqrt{t} \approx 1.26$$

$$N(d_1^B) = \Delta = \Pr(Z \leq d_1^B), Z \sim N(0;1) \approx 0.996$$

$$N(d_2^B) = \Pr(Z \leq d_2^B) \approx 0.8971.$$

Substituting the obtained results in the Black-Scholes formula we get:

$$c_B = S_B N(d_1^B) - X_B e^{-rt} N(d_2^B) \approx \$234.118m.$$

Thus, company B initially has approximately \$234.118m of equity and \$15.882m of debt outstanding in its capital structure.

Assume that A's share in the combined firm's value is $w_A = 30$ per cent = 0.3 and the remaining part comes to B so $w_B = 1 - w_A = 0.7$. Thus the value of merged firm C (S_C) = $w_A S_A + w_B S_B = w_A S_A + (1 - w_A) S_B$:

$$S_C = 0.3 * \$200m + 0.7 * \$250m = \$235m.$$

Given no additional debt-issues prior to the merger the FV value of the C's debt (X_C) = $X_A + X_B = \$130m + \$70m = \$200m$ in zero-coupon bond that will mature in 12 years (i.e. t in case of firm C = 12).

At our next step, we should estimate the volatility in the value of firm C (σ_C). The variance in the merged firm's value equals σ_C^2 :

$$\sigma_C^2 = \text{Var}[S_C] = \text{Var}[w_A S_A + w_B S_B]. \quad (2.9)$$

Given the variance property we get:

$$\text{Var}[S_C] = \text{Var}[w_A S_A] + \text{Var}[w_B S_B] + 2\text{Cov}[w_A S_A; w_B S_B],$$

where:

$$\text{Var}[w_A S_A] = w_A^2 \text{Var}[S_A] = w_A^2 \sigma_A^2$$

$$\text{Var}[w_B S_B] = w_B^2 \text{Var}[S_B] = w_B^2 \sigma_B^2$$

$$\text{Cov}[w_A S_A; w_B S_B] = w_A w_B \text{Cov}[S_A; S_B].$$

Substituting for $\text{Cov}[S_A; S_B]$ (or $\sigma_{AB} = \rho_{AB} \sigma_A \sigma_B$) we obtain the following expression for $\text{Var}[S_C]$ (see formula 2.10 for the variance of merged firm's value):

$$\text{Var}[S_C] = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B. \quad (2.10)$$

Estimation results yield $\text{Var}[S_C] = \sigma_C^2 = 0.07$ and thus the volatility in the value of the combined firm C (σ_C) = $\sqrt{\sigma_C^2} \approx 0.26 = 26$ per cent, which is even lower than volatility in the firm's A value (σ_A) of 0.3. The reason for such a low value is the negative correlation coefficient (ρ_{AB}) of -0.3 , although relatively small in absolute value.

We can note that in Damodaran's example, substantiating his suggestion of mergers' adverse effect of on equity, the value of the combined firm C (S_C) was calculated as an aggregate of the ones of firms A and B, thus $S_C = S_A + S_B$ so the value of merged firm incorporates 100 per cent of both S_A and S_B (i.e. w_A and w_B parameters are equal to 1 and thus can't be treated as corresponding weights of firms' A and B values in one of the combined firms that by definition should be greater than zero but smaller than one and sum into unity. In this case, the formulas (2.9) and (2.10) for the variance and the volatility in the combined firm's value (i.e. σ_C^2 and σ_C correspondingly) should be reduced to the following form:

$$\sigma_C^2 = \text{Var}[S_C] = \text{Var}[S_A + S_B] = \text{Var}[S_A] + \text{Var}[S_B] + 2\text{Cov}[S_A; S_B] \quad (2.11)$$

$$\sigma_C^2 = \sigma_A^2 + \sigma_B^2 + 2\rho_{AB} \sigma_A \sigma_B$$

$$\sigma_C = \sqrt{\sigma_C^2} = \sqrt{\sigma_A^2 + \sigma_B^2 + 2\rho_{AB} \sigma_A \sigma_B}.$$

However, Damodaran used formula (2.9) for σ_C^2 thus assuming:

$S_C = w_A S_A + w_B S_B$, where $w_B = 1 - w_A$ that itself contradicts the fact that merged firm's value was initially calculated as $S_C = S_A + S_B$.

The corresponding weights were also estimated inappropriately as:

$$w_A = \frac{S_A}{S_C}$$

$$w_B = \frac{S_B}{S_C} = 1 - w_A, \text{ since } S_C = S_A + S_B.$$

Given such weights' interpretation we have:

$$\begin{aligned} \left. \begin{aligned} S_A &= w_A S_C \\ S_B &= w_B S_C \end{aligned} \right\} \Rightarrow S_C = S_A + S_B = \\ = w_A S_C + (1 - w_A) S_C = S_C \text{ since } w_B = 1 - w_A. \end{aligned} \quad (2.12)$$

Obviously, the expression (2.12) above is not equivalent to the following one:

$$S_C = w_A S_A + w_B S_B.$$

However, it was treated as such in Damodaran's example and so the obtained parameters w_A and w_B were used as corresponding weights in formula (2.9) for σ_C^2 , thus leading to an erratic estimate of volatility in the value of the merged firm C (σ_C).

Given this fact, all the further obtained results cannot be trusted, so I decided to design my example that at least would not be subject to such contradictions to either prove or instead reject Damodaran's suggested hypothesis. From (2.9) it is clear that $\text{Var}[S_C]$ is positively related to ρ_{AB} and so is the σ_C being the square root of $\text{Var}[S_C]$. Formally this can be shown, considering the derivative of $\text{Var}[S_C]$ with respect to ρ_{AB} :

$$\begin{aligned} \frac{\partial \text{Var}[S_C]}{\partial \rho_{AB}} &= 2w_A w_B \sigma_A \sigma_B > \\ &> 0, \text{ since } w_A, w_B, \sigma_A, \sigma_B > 0 \end{aligned}$$

Hence both $\text{Var}[S_C]$ and σ_C will increase with ρ_{AB} . Thus since $\rho_{AB} \in (-1; 1)$ then if the value of firms A and B were highly or even perfectly positively correlated ($\rho_{AB} = 1$) the resulting volatility in combined firm's value (σ_C) would be much higher, and thus the value of its equity would also increase as compared to the case of $\rho_{AB} = -0.3$. Mathematically we have:

if $\rho_{AB} = 1$, then:

$$\begin{aligned} \sigma_C^2 &= \text{Var}[S_C] = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + \\ &+ 2w_A w_B \sigma_A \sigma_B = (w_A \sigma_A + w_B \sigma_B)^2 \end{aligned}$$

$$\begin{aligned} \sigma_C &= \sqrt{(w_A \sigma_A + w_B \sigma_B)^2} = |w_A \sigma_A + w_B \sigma_B| \\ &\text{given } w_A, w_B, \sigma_A, \sigma_B > 0: \end{aligned}$$

Table 6
Information on firms' capital structure

	A	B	A + B	C (30% A + 70% B)
Equity	\$170.249m	\$234.118m	\$404.367m	\$188.942m
Debt outstanding	\$29.751m	\$15.882m	\$45.633m	\$46.058m
Total	\$200m	\$250m	\$450m	\$235m

Source: The author.

$$\sigma_C = w_A \sigma_A + w_B \sigma_B \approx 0.37 > 0.26 \text{ (i.e. } \sigma_C \text{ given } \rho_{AB} = -0.3).$$

Since the debt maturity period (t) stays the same, the corresponding risk-free rate (r) should not be changed as well. Having obtained all the necessary inputs, we can now substitute them into the Black-Scholes model to find the value of the merged firm's equity and outstanding debt.

$$S_C = \$235m$$

$$X_C = \$200m$$

$$d_1^C = \frac{\ln\left(\frac{S_C}{X_C}\right) + \left(r + \frac{\sigma_C^2}{2}\right)t}{\sigma_C \sqrt{t}} \approx 2.23$$

$$d_2^C = d_1^C - \sigma_C \sqrt{t} \approx 1.33$$

$$N(d_1^C) = \Delta = \Pr(Z \leq d_1^C), Z \sim N(0;1) \approx 0.987$$

$$N(d_2^C) = \Pr(Z \leq d_2^C) \approx 0.908$$

$$c_C = S_C N(d_1^C) - X_C e^{-rt} N(d_2^C) \approx \$188.942m.$$

Thus, the combined company C will have nearly \$188.942m of equity and thus about \$46.058m of debt outstanding. Let us summarize the obtained results on the firms' capital structure in the table for the ease of comparison (see Table 6).

Based on the information from the table above the aggregate value of equity before the merger (i.e. A + B) is \$404.367m and it will substantially drop by more than a half (i.e. by \$215,425m) up to \$188.942m in merged firm C. At the same time, the level of outstanding debt, in contrast, will rise as a result of a merger by nearly \$0.425m from the total aggregate amount of \$45.633m to \$46.058m. Hence the wealth of bondholders will improve at the expense of the stockholders if the merger takes

place. This occurs partly to a negative correlation between merging firms' free cash flows that yields the relatively small volatility in combined firm's value and thus implies the lower value of equity and as a result the higher level of outstanding debt as compared to the ones of firms A and B when taken in aggregate. Thus, conglomerate mergers not followed by increases in leverage indeed may result in a wealth transfer from equity holders to debt holders.

As we have seen real options could be referred to as an asset (particularly intangible one that generally cannot be valued by traditional DCF approach) or extended investment opportunity providing its holder with the right to receive additional cash flows under certain circumstances, such as an option to delay, expand or abandon a particular project.

Real options may be valued using standard option pricing approaches, including the Binomial model and the Black-Scholes model. However, the latter is applied more frequently due to its relative simplicity and generality, enabling them to undertake rather complicated valuations without turning them into a time-consuming, exhausting process.

Despite some limitations mainly implied by its underlying assumptions, the real-option technique provides its user with the extended flexibility and thus has a broad scope of application in investment, asset or equity valuation and even may be extended to value the entire firm under the corresponding assumptions.

Given particular circumstances this approach may be either applied as a complement to traditional techniques so as to account for the potential value-adding factors generally neglected by standard DCF model or in contrast be the only possible valuation tool, thus turning it into a powerful practical instrument of value analyses that definitely should not be ignored.

Valuing Patent Rights of Amgen Inc. on “Parsabiv” and Determining the Optimal Time of its Exercise

Problem Overview

In this part, we will consider the application of the real-option approach in the determination of the optimal time of the patent. The object of our analysis will be the recent patent of Amgen Inc. on the production and sale of Etelcalcetide (trade name “Parsabiv”). Amgen Inc. is one of the leading companies in U.S. biotech-industry that develops manufactures and implements innovative drugs based on genetic engineering. Founded in 1980, Amgen is known as the leader in its industry sector, as it was among the first biotech-firms that managed to unleash the potential of a new generation of effective and safe drugs to provide patients with innovative methods of serious diseases treatment.

Etelcalcetide is a calcimimetic drug for the treatment of secondary hyperparathyroidism in patients undergoing hemodialysis. Initially, the drug’s formula was developed by another company KAI Pharmaceuticals that consequently held the patent on it. However, based on the information from the company’s official press release² Amgen Inc. acquired KAI in 2012. Thus, it could be assumed that in 2013 the acquisition process had been already finished and thus Amgen obtained the patent rights on Etelcalcetide, expiring at the end of 2030.

According to the information from the U.S. National Library of Medicine³ the drug was synthesized in 2013, meaning that Amgen exercised the patent on Etelcalcetide in the year just immediately following its acquisition.

However, although having exercised the patent and thus having the drug developed, Amgen could not market its product Parsabiv (trade name for Etelcalcetide) until its approval by the US Food and Drug Administration (USFDA) for treatment

of secondary hyperparathyroidism (HPT) in adult patients with chronic kidney disease (CKD) on hemodialysis in 2017. Thus, Amgen did not receive any cash flows from its product sales until that time.

Was it the right decision to exercise a patent immediately or it would have been better to wait till later times and if yes what would be the optimal time of converting a patent in a commercial product? These are the issues considered in this chapter.

Patent Value Estimation

We will begin our analysis from estimating the value of Amgen’s patent on Parsabiv using the technique described in part 2 based on the application of the modified Black-Scholes model. According to this approach, the value of the patent will be equal to one of the corresponding options to delay the drug development project provided by the patent. Thus, we need to obtain the necessary data on modified Black-Scholes model’s required inputs to get the answer.

It should be noted that we will estimate the patent’s present value as at the year of 2013. Thus in 2013 the corresponding real option to delay had 17 years remained till its expiration since Amgen owns the patent rights on Parsabiv production and marketing till the end of 2030 (i.e. $t = 17$).

Given the information above Amgen exercised the patent on Parsabiv in 2013 and thus invested the amount X in its development. The estimation of this investment’s amount is probably the toughest thing to do in this section since there is no publicly available information on this variable thus we used the average figure based on the information on research and development (R&D) spending statistics for pharmaceutical companies as at 2013 provided by Astra Zeneca company (Al-Huniti, 2013, p. 23) and therefore assumed $X = \$3,692.14\text{m}$.

At our next step, we calculated the present value (as at 2013) of the corresponding cash flows to be received from the Parsabiv’s sales using the historical data obtained from the official company’s financial statements and from Bloomberg Terminal and also forecasting it for future periods till the end of 2030 mostly assuming stable average growth rates of Amgen’s financial indicators and relevant ratios.

² FDA Approves Amgen’s Parsabiv™ (Etelcalcetide), First New Treatment in More than a Decade for Secondary Hyperparathyroidism in Adult Patients on Hemodialysis. Available at: <https://www.amgen.com/media/news-releases/2017/02/fda-approves-amgens-parsabiv-etelcalcetide-first-new-treatment-in-more-than-a-decade-for-secondary-hyperparathyroidism-in-adult-patients-on-hemodialysis>.

³ U.S. National Library of Medicine National Center for Biotechnology Information. Etelcalcetide. Available at: <https://pubchem.ncbi.nlm.nih.gov/compound/Velcalcetide>.

Each cash-flow from Parsabiv was estimated as the share of the corresponding year's net operating profit (i.e. EBIT(1 – tax)) from product sales — this share was assumed to be equal to one of Parsabiv's sales in a total amount of company's product sales. Since Parsabiv is a relatively new product, it will at first lack public trust at the primary stage of its introduction, which thus results in its initial share in total sales to be rather insignificant. This may also be explained by the company's intention no to produce large amounts of new product in order not to incur additional losses in case of low product demand. Thus, the initial quantity produced will be limited, and so will the primary product sales, undertaken at most to test the market conditions so as to determine the further sales development strategy.

However, given the innovativeness of Parsabiv and according to forecasted estimates provided by Bloomberg the first stage of introduction will take only several years and then its sales' share are assumed to grow relatively fast up until the year of 2023, so by that time this share will slightly exceed the one of Amgen's another drug — Sansipar which is a close substitute of Parsabiv being the less advanced predecessor of the latter. After that, Parsabiv's share in total product sales is expected to experience the normal average growth rate corresponding to the one of Sensipar. Obviously, this assumption makes our forecast rather conservative — however, given the prudence accounting concept, it is better to understate income rather than expenses.

Effective tax rates were also assumed to be subject to stable average growth, estimated without considering the abnormal fluctuations due to accounting adjustments to recent Tax Act enacted by the US on December 22, 2017. At the same time, we expect the tax rate to be no more than the US current corporate income tax of 21 per cent. It is likely that the Amgen's effective tax rate will be generally lower than the corporate one due to tax reliefs provided by US government to companies involved in R&D. However, we again chose the conservative approach on prudence grounds to be on the safe side.

The obtained cash flows were discounted at Amgen's WACC using its historic values obtained from Bloomberg Terminal and assuming it constant in future periods given its insignificant average growth rate.

As a result of our calculations, we have obtained the following discounted cash flows (DCF)

for each period of Parsabiv's sales, as shown in the tables below.

Thus, the present value of cash flows from the Parsabiv's development (PV) amounts to approximately \$7627.548m, assuming them to be received starting from the next year after the investment in the product's development was made and thus the patent was exercised. However, the fact that Amgen couldn't market its newly synthesized drug after its development in 2013 until the approval of US FDA was received in 2017 allowing for the first sales to take place creates the development lag (dlag) of 3 years, that should also be taken into account in the estimation of the present value of cash flows from the Parsabiv's sales. Given that the corresponding cash flows are received evenly throughout the patent expiration period (t) the annual cost of the product's development

delay could be calculated as $y = \frac{1}{t} = \frac{1}{17} \approx 0.059$.

Therefore, PV of cash flows from Parsabiv's realisation adjusted to the development lag (S) was estimated as follows:

$$S = \frac{PV}{(1+y)^{dlag}} \approx \$6425.607m$$

Given the remaining patent life of 17 years, we take assumed the risk-free rate (r) to be equal to the one of the 20-year US Treasury bond in 2013 (i.e. $r = 0.0312$). Due to the lack of historical data on Parsabiv, being a relatively new product, the volatility in its cash flows (σ) was approximated by the one in cash flows received from the sales of its close substitute Sansipar (i.e. $\sigma \approx 0.226$). Having obtained the key inputs, we can now estimate the other parameters of the modified version of Black-Scholes model and then estimate the value of the patent, which coincides with the one of the corresponding real call option to delay:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \approx 0.56$$

$$d_2 = d_1 - \sigma\sqrt{t} \approx -0.38$$

$$N(d_1) = \Delta = \Pr(Z \leq d_1), Z \sim N(0;1) \approx 0.7113$$

$$N(d_2) = \Pr(Z \leq d_2) \approx 0.3534.$$

Table 7
Parsabiv's DCF calculation output

	31.12.2017	31.12.2018	31.12.2019
Year N° (i)	1	2	3
	\$	\$	\$
Product sales	21795000320	22532999168	21881339661
EBIT(adj.)/Sales, %	59	58	59
EBIT (adj.)	12858000640	13116999424	12967750935
Eff. Tax rate (tax)	0.794	0.121	0.206
EBIT (1-tax)	2648748132	11529842494	10302053293
Parsabiv's share in total sales, %	0.023	1.491	2.429
CF (Parsabiv)	607650,3999	171926828,2	250285019,8
WACC, %	10	8	8
DF	0.907	0.852	0.787
DCF	551406.8964	146556990.9	196983200.4

Source: The author.

Table 8
Parsabiv's DCF calculation output

	31.12.2020	31.12.2021	31.12.2022
Year N° (i)	4	5	6
	\$	\$	\$
Product sales	22005103514	22976624931	23991038847
EBIT(adj.)/Sales, %	60	61	63
EBIT (adj.)	13276678018	14113264999	15002566808
Eff. Tax rate (tax)	0.21	0.21	0.21
EBIT (1-tax)	10488575634	11149479350	11852027778
Parsabiv's share in total sales, %	3.094	4.490	6.517
CF (Parsabiv)	324498464	500640503.6	772394761.9
WACC, %	8	8	8
DF	0.727	0.671	0.619
DCF	235797079.7	335879272.2	478440554.3

Source: The author.

Therefore, the corresponding call value is:

$$c = Se^{-yt} N(d_1) - Xe^{-rt} N(d_2) \approx \$913.525m.$$

Hence the Amgen's patent on Parsabiv approximately worth \$913.525m.

Determination of Optimal Time to Exercise the Patent Rights

Was it the right decision to exercise a patent immediately or Amgen should have waited till later times? Since the NPV from the Parsabiv's devel-

opment in 2013 equals $S - X \approx \$2733.47m$, so that it exceeds the obtained value of the patent (c) then for Amgen it's definitely better to convert Parsabiv into the commercial product as soon as possible than to hold the patent on it that will become less valuable with each year of the development delay that will shrink the period of the patents expiration (t) and the same time will increase annual costs of delay (y) thus leading to a decrease in the corresponding call's value. Hence the Amgen's decision to develop the Parsabiv in 2013 was reasonable.

Table 9
Parsabiv's DCF calculation output

	31.12.2020	31.12.2021	31.12.2022
Year N° (i)	4	5	6
	\$	\$	\$
Product sales	22005103514	22976624931	23991038847
EBIT(adj.)/Sales, %	60	61	63
EBIT (adj.)	13276678018	14113264999	15002566808
Eff. Tax rate (tax)	0.21	0.21	0.21
EBIT (1-tax)	10488575634	11149479350	11852027778
Parsabiv's share in total sales, %	3.094	4.490	6.517
CF (Parsabiv)	324498464	500640503.6	772394761.9
WACC, %	8	8	8
DF	0.727	0.671	0.619
DCF	235797079.7	335879272.2	478440554.3

Source: The author.

Table 10
Parsabiv's DCF calculation output

	31.12.2023	31.12.2024	31.12.2025
Year N° (i)	7	8	9
	\$	\$	\$
Product sales	25050238956	26156202563	27310994267
EBIT(adj.)/Sales, %	64	65	66
EBIT (adj.)	15947905097	16952810825	18021037441
Eff. Tax rate (tax)	0.21	0.21	0.21
EBIT (1-tax)	12598845026	13392720552	14236619579
Parsabiv's share in total sales, %	9.458	10.003	10.578
CF (Parsabiv)	1191660810	1339636071	1505986257
WACC, %	8	8	8
DF	0.572	0.528	0.488
DCF	681510837.2	707356561.1	734182462.3

Source: The author.

However, given the conservative assumptions about the future growth in Parsabiv's sales share and tax rates underlying the calculation of the patent value it could have been the case that the obtained estimate (c) will instead exceed the corresponding project's NPV if the more optimistic course of action is considered instead. If it is true than holding Parsabiv as a patent would be preferable than converting it immediately into a commercial product. However, in this case, one will likely to be concerned with a question about the optimal period of holding it as a patent.

This issue was considered by Damodaran (2012, p. 1103) in his book on investment valuation on the example of a patent on drug Avonex owned by another the US biotech firm Biogen, which value (c) estimated by the similar real-option technique appeared to be higher than the NPV from its immediate conversion into the commercial product in 1997 (i.e. development of drug Avonex and thus the exercise of the patent rights on it) with the corresponding difference being the time premium for holding a patent on Avonex rather than investing in its development as a product.

Table 11
Parsabiv's DCF calculation output

	31.12.2026	31.12.2027	31.12.2028
Year N° (i)	10	11	12
	\$	\$	\$
Product sales	28516769819	29775780148	31090375561
EBIT(adj.)/Sales, %	67	68	70
EBIT (adj.)	19156574908	20363664600	21646815150
Eff. Tax rate (tax)	0.21	0.21	0.21
EBIT (1-tax)	15133694177	16087295034	17100983969
Parsabiv's share in total sales, %	11,187	11,831	12,511
CF (Parsabiv)	1692993086	1903221610	2139555398
WACC, %	8	8	8
DF	0.450	0.416	0.384
DCF	762025713.2	790924895.9	820920055.7

Source: The author.

Table 12
Parsabiv's DCF calculation output

	31.12.2029	31.12.2030
Year N° (i)	13	14
	\$	\$
Product sales	32463010127	33896246266
EBIT(adj.)/Sales, %	71	72
EBIT (adj.)	23010819288	24460771741
Eff. Tax rate (tax)	0.21	0.21
EBIT (1-tax)	18178547238	19324009675
Parsabiv's share in total sales, %	13,231	13,992
CF (Parsabiv)	2405236090	2703907855
WACC, %	8	8
DF	0.354	0.327
DCF	852052756.7	884366139.1
		\$
$PV = \sum DCF_i$		7627547926

Source: The author.

In this case, as argued by Damodaran the optimal time of patent exercise is the one at which this time premium will turn to zero (i.e. when $c = S - X$).

This point in time could thus found graphically by valuing the call assuming that all Black-Scholes model's key parameters other than patent life (t)

stay constant and saving the obtained estimates for each t to plot them further on the graph together with the current product's NPV. This can be treated as a simulation of the patent's early exercise at different times till its expiration period (thus allowing it to be presented as an American call) with the obtained values being its values

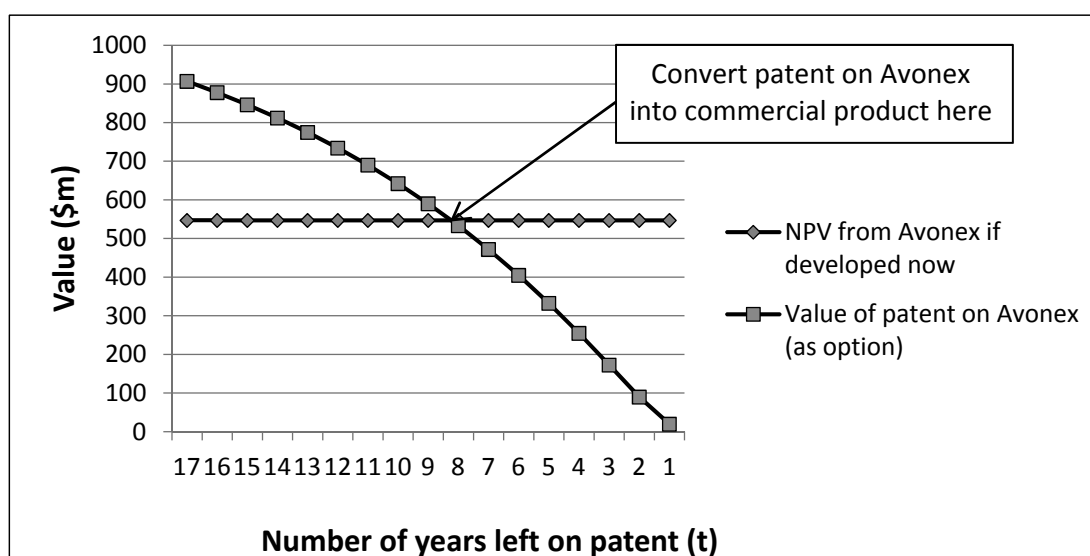


Figure 5. Patent value Vs. NPV (Avonex case)

Source: The author.

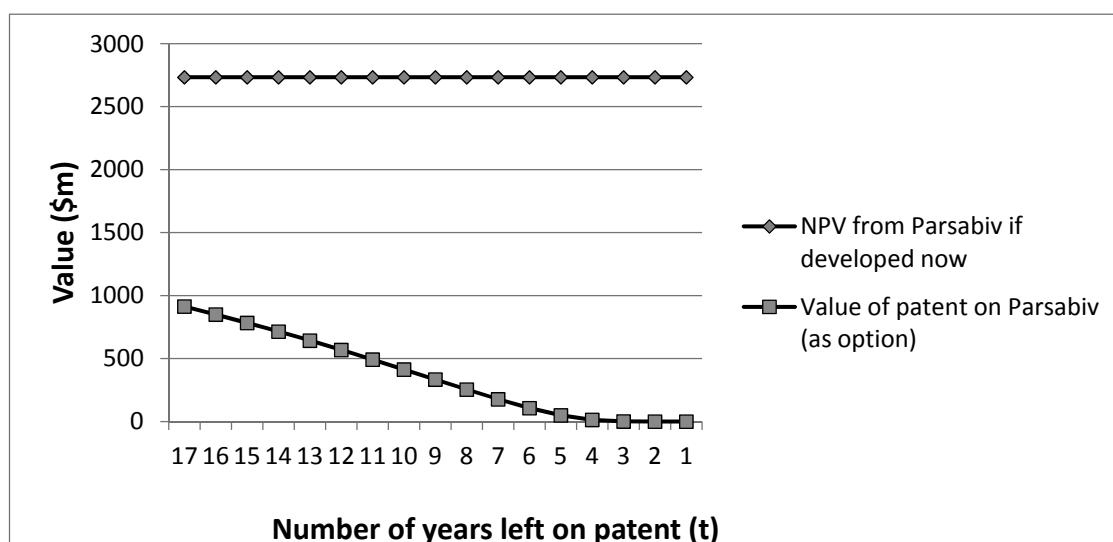


Figure 6. Patent value Vs. NPV (Parsabiv case)

Source: The author.

given the corresponding exercise time since the traditional Black-Scholes model doesn't allow for such flexibility to be taken into account and thus assumes that patent may be exercised only at the end of its life (i.e. a patent is assumed to be a real call option of European type). The optimal time to exercise is then determined as the one at which these two curves intersect.

Since with each year of product development delay, the patent life (t) will become shorter resulting in a higher cost of annual development delay that will yield lower call (i.e. a patent) values the curve illustrating the value of the patent as an option at different times will be therefore downward sloping.

Following this logic and using the initial data from the example in Damodaran's book we have obtained the graph of c (i.e. the value of the patent as an option) with respect to remaining patent life (t) plotted together with line fixed at the current level of NPV resulted from the immediate development of Avonex (the picture of the same graph was given in Damodaran's book — however, we decided to model it ourselves so as to ensure the understanding of the described methodology).

Moving back to our problem, we can plot the same graph for the case of Amgen's Parsabiv, illustrated in Figure 6.

Therefore given that the patent value increases with its remaining life, likely, the optimal time to

exercise the patent on Parsabiv (denoted as the point of intersection of two curves) had passed long before the time when Amgen Inc. acquired the KAI Pharmaceuticals and thus consequently became the owner of the corresponding patents rights.

Obviously, since the described technique of patent's optimal exercise time determination is based on simulation, it is not expected to yield precise results as compared to ones based on models applicable to American option valuation such as Binomial one or using the Black-Sholes equation with a non-linear function. However, these approaches are far more complicated and time-consuming. Thus, in a world when the time on decision-making is always limited the accuracy advantage is vanished, making such simulation the best tool providing with the general picture of overall value trends.

Conclusions

Asset valuation plays a significant role in various economic relationships, being a crucial element of any investment decision-making process and corporate valuations in general.

Classical approaches to asset valuation include several methods concerning different aspects of this process, each of which has its strengths and weaknesses.

However, given the availability and predictability of data on the corresponding variables such as future cash flows and discount rates the DCF approach is generally considered as the most accurate and superior traditional asset valuation technique since it takes into account the potential income the asset is expected to generate each future period of its remaining useful life with regard to the time value of money.

Real options are a response to the disadvantages and limitations of classical methods of asset valuation and assessment of investment projects that do not allow considering the decision elasticity inherent in them. Thus, the essence of real options is not to replace classical methods, but to supplement and enrich them with additional elements, often playing the ultimate role in the investment decision-making process.

Despite some limitations mainly implied by its underlying assumptions, the real-option technique provides its user with the extended flexibility and thus has a broad scope of application

in investment, asset or equity valuation and even may be extended to value the entire firm under the corresponding assumptions.

Given particular circumstances this approach may be either applied as a complement to traditional techniques so as to account for the potential value-adding factors generally neglected by classical approaches or in contrast be the only possible valuation tool, thus turning it into a powerful practical instrument of value analyses that definitely should not be ignored but instead is to be adopted by the analysts and other expert dealing with valuation problems so as to ensure the adequacy and comprehensiveness of the obtained estimates and thus not to make wrong decisions.

However despite the advantages of the extended flexibility in asset and investment project valuation, real options can be costly to obtain (e.g., the right to extend a lease or purchase a property), complex to value, and dependent on problematic assumptions — these are the main drawbacks of this approach. In this case, they should not be pursued unless the firm has the resources to exploit the option, and they add significantly to the value of the firm.

This research may be further developed to analyse the application of American option pricing models in corporate valuations.

By analogy with equity valuation in troubled firms, the real-options approach may be extended to access the fair value of countries where the amount of internal and external debt obligations overhang the value of the countries assets since the traditional methods can't be used in this case.

Real-option valuation technique may also be applied in the measurement of megapolises' potential boundaries. The intuition is as follows. According to the statistical data, the population in megacities is growing at a faster rate in comparison with general population growth. The main reason for this paradox is the internal migration of people from provinces to megapolises in search of potentially available new opportunities and better living standards. However, the authorities of megacities are aware of the fact the endless extensive growth of cities creates an array of problems; therefore, the supply of new housing development intentionally lags the existing demand for it. The amount of supply deficit in the land market is even more significant. All these factors lead to an increase in numbers of floors in

the constructed buildings as well as raise the price level in the real estate market, given the growth in construction costs due to rising land prices and the need to balance demand with limited construction opportunities.

Given these facts, such migration to megacities may be defined as a real option of people from provinces to receive new potential opportunities, so to attain better living conditions. Hence the migration to megapolises will continue as long as its value (as an option) remains positive. From an economic point of view, this option value will be positive unless the excess of the price level in megalopolises over the price level in provinces will exceed the number of new potential opportunities, provided by megacities to people migrating from provinces. Given the option nature of migration,

the government can thus not only predict by also regulate its flows by altering these parameters.

It should be noted that such migration to megacities a characteristic feature of all megacities all over the world. However, the time when it reaches its peak differs. Thus, the megacities in developed countries have already passed this stage, while the opposite is true for the ones in developing countries.

Therefore, this example reflects the extensive possibilities of real options approach applies not only in corporate valuations but also in the field of planning for the development of megapolises particularly in the context of forecasting the magnitude of internal migration providing room for estimation and control of the potential level of extension of city areas.

References

- Al-Huniti, Nidal. (2013). Quantitative Decision-Making in Drug Development. AstraZeneca.
- Angelis D.I. (2000). Capturing the options value of R&D. Research Technology Management.
- Bauer, M. (2009). An analysis of the use of discounting cash flow methods and real options to value flexibility in real estate development projects, Lambert Academic Publishing, Berlin.
- Black, F., Scholes, M. (1973). The pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637–654.
- Branch, Marion A. (2003). Real Options in Practice. New York: Wiley.
- Brennan, J., Schwartz, E. (1985). Evaluating Natural Resource Investments. *The Journal of Business*, 58(2), 135–157.
- Brennan, M. J., Trigeorgis, L. (2000). Project Flexibility, Agency, and Competition. Oxford: Oxford University Press.
- Bruslanova, N. (2004). Evaluation of investment projects using real options technique. *Finansovyy director*, 4, 20–23. (in Russian)
- Bulan, L., Mayer, C., Tsurriel, Somerville C. (2009). Irreversible investment, real options, and competition: evidence from real estate development. *Journal of Urban Economics*, 65, 237–251.
- Capozza, D., Li, Y. (1994). The intensity and timing of investment: the case of land. *The American Economic Review*, 84(4), 889–904.
- Childs, P. D., Riddiough, T. J., Triantis, A. J. (1996). Mixed uses and the redevelopment option. *Real Estate Economics*, 24, 317–339.
- Cox, J., Ross, S., Rubinstein, M. (1979). Options Pricing: a simplified approach. *Journal of Financial Economics*, 7, 229–263.
- Cox, Ross, Rubinstein. (1979). Option Pricing: A simplified approach. *Journal of Financial Economics*, 7, 229–263.
- Damodaran, Aswath. (2006). Damodaran on Valuation: Security Analysis for Investment and Corporate Finance. John Wiley & Sons.
- Damodaran Aswath. (2012). Investment Valuation: Tools and Techniques for Determining the Value of any Asset. University Edition. Wiley Finance Series.
- DePamphilis, D. (2017). Mergers, Acquisitions, and Other Restructuring Activities: An Integrated Approach to Process, Tools, Cases, and Solutions. Academic Press.
- Dixit, A. K., Pindyck, R. S. (1994). Investment under Uncertainty. Princeton, NJ: Princeton University Press.
- Dixit, A. K. (1992). Investment and hysteresis. *Journal of Economic Perspectives*, 6, 107–132.
- Folta, T. B., Miller, K. D. (2002). Real options in equity partnerships. *Strategic Management Journal*, 23, 77–88.
- Geske, R. (1979). The valuation of compound options. *Journal of Financial Economics*, 7(1), 63–81.
- Grenadier, S. R. (1995). Valuing lease contracts a real-option approach. *Journal of Financial Economics*, 38, 297–331.
- Hertz, D. (1964). Risk Analysis in Capital Investment. *Harvard Business Review*, 42, 95–106.

- Horn, A. et al. (2015). The Use of Real Options Theory in Scandinavia's Largest Companies. *International Review of Financial Analysis*, 41(C), 74–81.
- Howell, S. et al. (2001). Real Options. Evaluating Corporate Investment Opportunities in a Dynamic World. London: Financial Times Prentice Hall.
- Huchzermeier, A., Loch, C. (2003). Evaluating R&D Projects as Real Options: Why More Variability is not always better. Working Paper, WHU Koblenz and INSEAD.
- Kalinin, D. (2003). How to determine the “real” value of the investment project? An example of using the method of real options. *Rynok tsennykh bumag* [Securities market], 24, 62–65.
- Karpova, N., Pocherin, N. (2000). Application of option theory in determination of value of R&D and license agreements. *Voprosy otsenki* [Assessment issues], 2, 57–66.
- Krukovskiy, A. A. (2008). The method of real options in investment management. *Trudy ISA RAN* [Proceedings of ISA RAS], 37, 122–143.
- Kulatilaka, N., Markus, A. J. (1992). Project valuation under uncertainty: where does DCF fail? *Journal of Applied Corporate Finance*, 5(3), 92–100.
- Leslie, K. J., Michaels, M. P. (1998). Making real options real. *The McKinsey Quarterly*, 3, 128–141.
- Leslie, K. J., Michaels, M. P. (2000). The real power of real options. *The McKinsey Quarterly*, 3, 4–22.
- Leung, B.Y.P., Hui, E.C.M. (2002). Option pricing for real estate development: Hong Kong Disneyland. *Journal of Property Investment & Finance*, 20(6), 473–496.
- Limitovskiy, M. A. (2017). *Investitsionnyye proyekty i real'nyye optionsy na razvivayushchikhsya rynkakh: ucheb. posobiye dlya bakalavriata i magistratury* [Investment projects and real options in emerging markets: study guide for undergraduate and graduate students], 5th edition. Moscow: Yurayt Publ.
- Lucius, D. I. (2001). Real options in real estate development. *Journal of Property Investment & Finance*, 19(1), 73–78.
- Majda, S., Pinduck, R. (1987). Time to build, Option Value and Investment Decisions. *Journal of Financial Economics*, 18, 7–27.
- Mao, Y., Wu, W. (2011). Fuzzy Real Option Evaluation of Real Estate Project Based on Risk Analysis. *Systems Engineering Procedia*, 1, 228–235.
- Mathews, S., Datar, V. (2007). A Practical Method for Valuing Real Options: The Boeing Approach. *Journal of Applied Corporate Finance*, 19(2), 95–104.
- Milne, A., Whalley, A. E. (2000). Time to build, option value and investment decisions: a comment. *Journal of Financial Economics*, 56, 325–332.
- Morano, P., Tajani, F. (2013). The transfer of development rights for the regeneration of brownfield sites. *Applied Mechanics and Materials*, 409–410, 971–978.
- Mun, J. (2006). Real Options Analysis. Tools and techniques for valuing strategic investments and decisions. John Wiley & Sons.
- Myers, Stewart C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2), 147–175.
- Pacheco-de-Almeida, G., Zemsky, P. (2003). The effect of time-to-build on strategic investment under uncertainty. *Rand Journal of Economics*, 34, 166–182.
- Pirogov, N. K., Salomykova, O. A. (2007). Analysis of companies' growth options in the emerging capital markets. *Korporativnyye finansy* [Corporate finance], 2(2), 32–42.
- Pirogov, N. K., Zubtsov, N. (2008). Interaction of real options on the example of development projects in Russia. *Korporativnyye finansy* [Corporate finance], 6(2), 40–55.
- Quigg, L. (1993). Empirical testing of real option-pricing models. *Journal of Finance*, 48(2), 621–39.
- Rocha, K., Salles, L., Alcaraz, Garcia, F. A., Sardinha, J. A., Teixeira, J. P. (2007). Real estate and real options — a case study. *Emerging Markets Review*, 8, 67–79.
- Schatzki, T. (2003). Options, uncertainty, and sunk costs: an empirical analysis of land use change. *Journal of Environmental Economics and Management*, 46, 86–105.
- Shen, J., Pretorius, F. (2013). Binomial option pricing models for real estate development. *Journal of Property Investment & Finance*, 31(5), 418–440.
- Shoup, D. (1970). The optimal timing of urban land development. *Regional Science Association Papers*, 25, 33–44.
- Stewart, B. (2002). The quest for Value. Harper Collins.
- Tan, Jackson J. (2018). Interfaces for enterprise valuation from a real options lens. *Strategic Change*, 27(1), 69–80.

- Titman, S. (1985). Urban Land Prices Under Uncertainty. *The American Economic Review*, 75(3), 505–514.
- Triantis, A. J., Hodder, J. E. (1990). Valuing flexibility as a complex option. *Journal of Finance*, 45, 549–565.
- Trideorgis, L. (2003). Real options: Managerial flexibility and strategy in resource allocation. Cambridge, Mass: MIT Press.
- Vonnegut, A. (2000). Real Options Theories and Investment in Emerging Economies. *Emerging Markets Review*, 1, 82–100.
- Vygon, G. V. (2004). Estimation of the fundamental value of oil fields: the method of real options. *Ekonomika i matematicheskiye metody* [Economics and mathematical methods], 37(2), 54–69. Data-sources: Bloomberg Terminal Drugbank. Etelcalcetide. Available at: <https://www.drugbank.ca/drugs/DB12865> (Retrieved 3 May 2019)
- FDA Approves Amgen's Parsabiv™ (Etelcalcetide), First New Treatment In More Than A Decade For Secondary Hyperparathyroidism In Adult Patients On Hemodialysis. Available at: <https://www.amgen.com/media/news-releases/2017/02/fda-approves-amgens-parsabiv-etelcalcetide-first-new-treatment-in-more-than-a-decade-for-secondary-hyperparathyroidism-in-adult-patients-on-hemodialysis> (Retrieved 3 May 2019)
- Official annual financial statements of Amgen Inc.
- U.S. National Library of Medicine National Center for Biotechnology Information. Etelcalcetide. Available at: <https://pubchem.ncbi.nlm.nih.gov/compound/Velcalcetide> (Retrieved 3 May 2019)

Оценка справедливой стоимости активов с использованием метода реальных опционов

Александр Гулабян

Бакалавр экономики факультета международных финансов, Департамент мировой экономики и международных финансов, Финансовый университет, Москва, Россия

Аннотация. Целью данной работы является анализ и систематизация возможных подходов к оценке реальных опционов, особенно при рассмотрении практических аспектов их применения в реальных оценочных задачах. Объектом исследования являются модели ценообразования опционов, а предметом — их применение при оценке реальных опционов, заложенных в корпоративные оценки, особенно с учетом побочных эффектов.

В статье делается попытка:

- сформулировать понятие справедливой стоимости и проанализировать традиционные подходы к ее расчету в контексте оценки активов;
 - определить реальный опционный подход к оценке справедливой стоимости и проанализировать его теоретические предпосылки;
 - определить роль подхода к реальным опционам в традиционной системе методов оценки стоимости.
- Проанализированы практические аспекты их применения в задачах оценки с учетом соответствующих примеров и приведены реальные примеры применения этой методики в современных рыночных условиях с использованием последних данных.

Ключевые слова: оценка реальных опционов; оценка активов; оценка справедливой стоимости; подход к реальным опционам