

# Using Elliott Wave Theory Predictions as Inputs in Equilibrium Portfolio Models With Views\*

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**Abstract.** We evaluate historical performance of one of the most famous Elliott Wave Theory proponents – Robert Prechter using Black-Litterman as framework for portfolio optimization with views. Our choice of the portfolio model for historical backtest contradicts to traditional "straightforward" approach to test historical predictions performance. We argue that this approach is more realistic as it allows to model Bayesian-rational decision-making of risk-averse agent with views, fueled by Elliott theory. Our results show that use of mentioned framework Elliott Wave Theory offers brings value to investor.

**Аннотация.** Мы оцениваем гипотетическую историческую доходность Роберта Пречтера, одного из самых известных сторонников волновой теории Эллиотта. Для этого мы используем модель оптимизации портфеля по Блеку-Литтерману. Наш подход противостоит традиционному подходу проверки исторической доходности предсказаний. Мы считаем, что такой подход является более реалистичным, так как позволяет моделировать рациональный метод принятия решений по Байесу для агента с суждениями, основанными на теории Эллиотта. Полученные с помощью такого подхода результаты показывают, что волновая теория Эллиотта имеет ценность для инвестора.

**Key words:** Black-Litterman portfolio optimization, Bayesian decision theory, Elliott Wave Theory, technical analysis.

## INTRODUCTION

Ability of market practitioners to outperform market by predicting prices was in the focus of academicians at least since Cowles's seminal papers "Can stock market forecasters forecast?" (Cowles, 1934), followed by many papers with almost identical approach to research but different results. In (Goetzmann, Brown, 1997) an attempt has been made to reconsider some of Cowles's results on a risk-adjusted basis, using advances of modern portfolio theory. Namely, authors find that timing strategy of one of the major proponents of the Dow Theory, William Peter Hamilton, generate both positive alphas and high Sharpe ratios. On this basis they conclude that there could be some value in Dow Theory, at list there was value in the version, which was used by Hamilton.

Elliott Wave Theory, or Elliott wave principle, is a quasi-theory, pretending to explain and predict returns of the stock market, developed by Ralph Nelson Elliott (1871–1948), and having much in common

with Dow Theory. Just as in case with the latter, Elliott Theory could be better perceived in narratives of its proponents, describing how they apply it in practice, other than works, trying to describe on more theoretical level how it should be applied. Softness, fuzziness and non-falsifiability of many of the theory predictions do not allow it to be usual scientific theory. From the other hand, narratives of Elliott Theory are often formulated in flexible form, which refuses direct historical simulation of the form "buy-hold-sell an asset". Predictions could be formulated in relative manner ("equities would outperform bonds"); cover groups of asset classes ("we believe capital assets to grow N%"); and even miss some asset classes in an irregular manner (i.e. mention class A and class B in time  $t_1$ , class A and class C in time  $t_2$ , class B and class C in  $t_3$ , etc.).

One of the advantages of Black-Litterman approach to asset allocation is that it enables to specify investor's views either in direct or relative form, and cover either single asset classes or groups of asset

\* Использование предсказаний волновой теории Эллиотта в моделях равновесных портфелей с суждениями.

classes. Following approach of (Cowles, 1934) and (Goetzmann, Brown, 1997) those views will be reconstructed from market predictions made by one of the most famous "guru" of Elliott Wave Theory, Robert Prechter, who began his professional career in 1975 as technical market specialist with the Merrill Lynch Market Analysis Department in New York. We reconstruct the whole picture of Prechter's views and its evolution using Elliott theory's rules based on his narratives published since 1979 in "The Elliot Wave Theorist, Global Market Perspective" newsletter. We further develop approach of (Goetzmann, Brown, 1997) and extend it to Bayesian decision-making in equilibrium market settings, represented by Black-Litterman framework.

The rest of the paper is organized as follows. Next two sections briefly survey Dow Theory performance as reflected in academic literature and Elliott wave principle as reflected in Prechter's theoretical works and narratives. Then we continue with Black-Litterman discussion and description of our backtest and summary of its results. In the final section we conclude.

## THE DOW THEORY

The Dow Theory is known to be the background for Elliott Wave Theory. Both typically are described in terms of investor sentiment and crowd behavior. Both Elliott and Dow developed their ideas long before computers were available, so there was little available in the way charts or easily computed technical indicators. The easiest way to describe any theory of stock market behavior would have been to draw on the investors' sentiment and traders who made up the market. The other major comparison between the Dow and Elliott Wave Theory is that both speak of waves. The Dow Theory refers to an accumulation phase, the period when traders who trade on technical analysis enter the market, and the final phase of run-up, when all the investing people enter a bull market. These may approximately coincide with the upward legs of a five-wave impulse move. The Dow Theory does have important point in common with Elliott, Dow Theory discusses primary, secondary, and minor trends in the market. This is the expression of the fractal nature of the markets in which larger trends subdivide into smaller actions and reactions. So it is appropriate to say that the Elliott Wave Theory is built upon this solid foundation.

Cowles in (Cowles, 1934) tried to test if the Dow Theory works properly, and came to conclusion that it does not work. The main reason for that were the outcomes he got after comparison of two strategies. He calculated total returns for Dow timing strategy

and got 12% of yield, whereas alternative investment in 100% stock market gave 15.5 % of return per annum. However, he neglected to adjust to relative risk. Cowles also conducted nonparametric analysis of Hamilton's recommendations which show the frequency of correct Bull and Bear market calls. The results were not satisfactory: 29 bullish, 23 bearish, 38 neutral, which means the same as flipping coins. As noted by (Goetzmann, Brown, 1997), the reason for his mistakes is that Cowles did not consider the repetitive Bull forecast in rising market and repeated bear forecast in falling market. There are many methods which could be applied to Dow Theory. Goetzmann, Brown argue that nonparametric Hendrickson-Merton test is the natural test for Dow theory. HM test quite effectively determines whether the manager provided put on the market when it was necessary. It is proper test for Hamilton since he uses the frequency of correct Bear market calls as the basis for verifying market success. The test confirms that HM test is effective in bear markets, so that proportion of correct Bear calls is higher than just in case flipping a coin.

Approach of Goetzmann, Brown is generally based on simulation of trading strategy that shifts from long stocks to short stocks. In comparison with Cowles who used 50 to 50 portfolio mixtures of Dow Industrials and Dow Railroads, the S&P index was used as basis. Short-term commercial paper rates represent alternative investment. Overall, results showed that Hamilton's portfolio is less risky than fully invested strategy. Results on simulated investment strategy were almost indistinguishable from average return holding S&P all stock portfolio.

Hamilton's editorial serves as rare chance to think and recover Dow theorist rules on making good market analysis. All evidences against Hamilton's timing ability seem not justified. Over 27 years' period of market observation with application of Dow Theory brought positive risk adjusted returns. Despite doubts whether it is luck or really good skill of market prediction, it seems that Hamilton followed rules based upon his observations of market trends which could be recovered by nonlinear estimation methods. The fact that Hamilton was successful market timer reasonably justified in the article; this is one of the values of this paper. Also the analysis conducted by Cowles on Hamilton's records led to the idea of random walk hypothesis and played significant role in establishment of efficient market theory.

## ELLIOTT WAVE THEORY

In the 1930-s, Ralph Nelson Elliott discovered the first known fractal mathematical model of market dy-

namics. He derived this model through observations of past and current market behaviour. The big part of the Wave principle consists of detailed description of various types of these patterns like five- and three-wave patterns and demonstrated the exact rules that should be followed when joining together to form larger pattern. The patterns he distinguished are repetitive in form, but not necessarily in time or amplitude. Unbreakable rules concerning five-wave movements are: (1) The second wave cannot carry past the beginning of the first wave, or simply second wave does not begin from the start of first wave; (2) The fourth wave cannot enter the price range of the first wave as a rule; (3) The third wave cannot be the shortest wave among other four waves.

The guidelines used to five-wave pattern are: (1) If wave two develops as one type of pattern, then wave four usually develops different type of pattern. Second waves are considered to be the strongest, relatively fast, steep retracements of the prior impulsive movements, whereas fourth waves tend to trace out complex, more sideways shape of wave; (2) Wave one among one, three and five is "extended" that is, it is the longest of the three movement and subdivides into component waves of nearly the size of the

other main waves. Wave three is usually extended. (3) When wave three is extended, wave one and five tend to be equal in their price movement and similar in complexity of internal structure. (4) Usually market movement tends to be related in size by a mathematical constant, the Fibonacci ratio and Golden mean, 0.618, and its complement 0.382.

If there is a violation of one or more of these rules, it implies that the wave count is incorrect. If the chart seems not like that, the sequence is not impulsive and Elliot Wave pattern cannot be used here — or there is an extension which will be discussed further. There is crucial point that should be pointed out and remembered. Although it is the best forecasting tool in existence, the Wave Principle is not primarily a forecasting tool: it is a detailed description of how markets behave. The primary value of the Wave Principle is that it provides a context for market analysis.

### RESTORATION OF PRECHTER'S PREDICTIONS USING ELLIOTT WAVE THEORY

For the purpose of finding returns on predictions made by Prechter based on Elliott Wave Theory, we

Date	Prediction	Interpretation
March, 1993	<i>E waves are always accompanied by extreme psychology, so bullish sentiment should be powerful by the time this rally peaks. After that peak, gold will experience its second and final decline to its ultimate bear market bottom."</i>	Triangle pattern still in place and we are on D wave of A-B-C-D-E. At the moment of forecast in March 1993 gold at the end of wave which says that bullish potential of E wave is coming soon. Dow value on March 1993 is 328. By applying Fibonacci projection and considering that wave E is strong wave, the next target value should be 385,400,408 which are correspond value of Fibonacci numbers. Since wave E considered to be corrective, it was divided to a-b-c corrective waves. What is interesting, tops of a-b-c corrective wave up exactly coincide with target values mentioned above 385,400,408. Wave E exited on august 1993 with the value about 408. Return received from forecast is the following: $ 328-410,5 /328/2=12,58\%$ Forecast was in place in two quarters.
May, 1998	<i>"As it turns out the percentage gain of wave V is 0.609 times that of waves I-III. This value is certainly is close to 0.618, 11889 but we will see".</i>	According to his writings, Prechter saw that 1982 bottom was the end of wave IV, and the impulse wave from that point can be counted as five-waves up. We can clearly see that the market rally in 1987 was counted as wave 2 down, and if we apply Fibonacci projection, we can see that the length of wave 1 was equal to 0.38 percent of the length applied to 1982 low . The next high reached at the beginning of 1998, and the down market from that point to 8800 is wave 4 down. Now we can see that we are in the wave 5 of Supercycle wave IV up.
February, 2000	<i>"In May 1998 we published that projections to 11889, which turned out to be only 1,5% away from the high recorded in January 2000 11722,98."</i>	Based on this prediction and its realization we can convert it to return by simple finding the difference between the point of Dow when prediction were made and forecasted point of realization dividing this difference by initial point of Dow when forecast was made and divide into period it took to realization (in our case it should be quarterly returns, we divide by 8, since 2 years*4 quarters).

Date	Prediction	Interpretation
		<p><math>(9055,14-11889)/9055,14/8]=3,91\%</math> quarterly return for each quarter since May 1998 till January 2000.</p> <p>As it was stated before, the high in January 2000 was the over of wave V of Supercycle Wave, as forecasted. The trend down from 2000 till 2002 could be counted as a corrective wave a-b-c expanded flat correction of previous high. The market downside trend from 2007 till 2009 was the c of expanded flat correction. Since that low market took five wave upside trend. In June 2012 we were in last fifth wave up which soon was going to exit and forecast made states that we should have major collapse lasting 4 years: Fifth wave should exit at the point about 15147,22 which is 0,618 of wave one up.</p>
June, 2004	<p><i>"This is in line with the wave structure on the monthly and quarterly charts. A third wave down is in progress from the recent high at 116<sup>11</sup>, and prices should continue to fall to 102<sup>23</sup>, potentially 101<sup>23</sup>/100<sup>06</sup>. Then in July, an upward fourth wave correction to 106/107 should lead to a new low in the third quarter between 100<sup>06</sup> and 97<sup>04</sup>. If prices follow this course, then the wave structure will be in place to support a rally to 115."</i></p>	<p>According to counting, the price is in the end of wave 2, so the bear market for bond is in place. Prechter states that there will be some wave 3 down to 100,06, and then go up for fourth wave correction and after that hard down to about 97. As in fact it is seen that the count is not right, since market went in opposite way. The negative return will give us <math>(97,13-111,15)/97,13/2</math> which is equal to <math>-7,22\%</math>. We constructed similar chart to Prechter's counting on the same historical chart, to show the opposite market. The difference between the point of forecast which is down and the point of real market value at that time divided by point of forecast. As you can see Elliott Wave Theory was counting work in real time in most cases. However, there are still cases when chart can be misinterpreted.</p>
January, 2006	<p><i>"Silver bulls are virtually certain that the recent steep decline is a great buying opportunity, which the high level of optimism argues is just the opposite. The first downside target is around the \$7.30 level, the apex of the triangle that ended in late August. Despite the strong bearish evidence, if the \$9.26 high is exceeded, silver's rise will probably end in a wild spike to significantly higher levels prior to a violent reversal. Any such event will not change downside targets. Silver 9,25."</i></p>	<p>Since 2004 silver had bullish triangle, which said that bullish market for silver is coming. In August 2005 this triangle had a bottom and started wave 1. In December 2005 counter wave 2 bottomed, which gave wave 3 to start. On the time of forecast, we have in 3 wave up with the price of 9,25. Application of Fibonacci ratio showed that since the wave 3 is the strongest and impulsive wave, it can go far away. The target was 14,36, however it went a little further. After this peak it sharply fell down, which says that wave 4 in place. The next target for wave 5 was 14.96, silver went slightly above that. The forecast has worked and gave return in <math>(9,25-15,22)/9,25/2=32,27\%</math>.</p>
January, 2006	<p><i>"Bonds broke beneath the lower channel line of the parallel trend channel formed by the rise from 110<sup>03</sup>. The decline to today's 112<sup>11</sup> low appears to be a clear impulse wave (five waves), which confirms that a top is in place. Bonds should now be declining back to, and likely well beneath the 110<sup>03</sup> low. The key point is that regardless if a near-term bounce develops, the trend has turned down and the bears control the bond market for the near term."</i></p>	<p>The picture says that after peak in mid of January, it started to go down and broke the trend channel up, which says that wave down began. We are at the end of wave 1 down, which says that some rallies up will be just corrective moves of five-wave pattern down. If we apply Fibonacci to June 2005 high and start of bigger wave down, whereas current market down is going to be the wave 5 down of bigger wave down. It shows that retracement of 38% down will lead to about 105. Return will be calculated as of the date December 2006, when the low was exactly 105,11. Return will be <math>(112,21-105,11)/112,21/2</math> which equals to 3,16% for two quarter period.</p>
January, 2006	<p><i>"Since 1980 The EWT has made a case that the gold has been tracing out a major bear market. The partial recovery pattern, which has retraced a Fibonacci 38,2% of wave W, is either ended December 1987 or an A_B_C_D_E triangle that requires one more rally."</i></p>	<p>Prechter interpreted market as being in bearish trend (probably in wave D down in case of triangle). So in August 1989 he is somewhere on fifth wave down of bigger wave D. Forecast expects a small thrust up if we apply Fibonacci retracement to 1987 year high. So we can conclude that expected wave up should retrace about 38% or 61% of 1987 high. In August 1989 Dow was at value 359,81, expected forecast is 410,5; we get return of 5,12% since from August 1989 till January 1990 three quarters.</p>
May, 2013	<p><i>"With the higher high on weaken momentum the stock market is far more vulnerable than at any time since Supercycle V end in 2000 year. The exhaustion depicted from recent t issue has led to a minor decline from Dow high 13338.7 on May 1. It should soon develop in major collapse lasting 4 years."</i></p>	<p>To calculate return we should do the same as in previous example. Dow value at time of forecast, June 2012, 12,566 and forecasted realized value is 15,275 in May 2013 divided by 4 quarters. We get about 7,19%.</p>

restore his countings and calculate projected returns based on information about restored Prechter's market views. Our analysis dates back to beginning of 80-s, and extends to the year 2013. In the following table we have summarized key predictions, made by Prechter, and our interpretation of it.

## BLACK-LITTERMAN MODEL

Black-Litterman model was firstly introduced in 1990 and further expanded in (Black, Litterman, 1991, 1992), (Bevan, Winkelmann 1998), (He, Litterman, 2002). It combined ideas of Markowitz' MVO (Markowitz, 1952), Sharpe's CAPM (Sharpe, 1964), reverse optimization (Sharpe, 1974), Theil's mixed estimation, and the universal hedge ratio of Black's global CAPM (Black, 1989). It was argued then that Black-Litterman model creates stable, mean-variance efficient portfolios, which are based on an investor's unique insights, and overcomes the problem of input-sensitivity. It avoids the problem of estimation error-maximization by spreading the errors throughout the vector of expected returns.

It starts with equilibrium no-views situation, which is useful in case Prechter had no views at all for some asset in some particular period. In the case there is an absence of view model just sticks to market views (implied excess equilibrium returns) obtained from reverse optimization process. Three main inputs are needed for calculation of implied excess returns: risk premium, covariance matrix and market capitalization of the assets. The vector implied excess equilibrium returns are derived from already available information applying the following equation:

$$\Pi = \lambda \Sigma w_{mkt} \quad (1)$$

$\Pi$  – the Implied Excess Equilibrium Return Vector (N x 1 column vector);

$\lambda$  – the risk aversion coefficient;

$\Sigma$  – the covariance matrix of excess returns (N x N matrix);

$w_{mkt}$  – the market capitalization weight (N x 1 column vector) of the assets;

Black-Litterman model assumes  $K$  represents the number of views and  $N$  demonstrates the number of assets. Some rearrangement of the previous formula by substitution  $\mu$  which characterizes any vector of excess return for  $\Pi$  which represent the vector of implied Excess Equilibrium Returns offers us new solution which can be considered as unconstrained maximization problem:

$$w = (\lambda \Sigma)^{-1} \mu \quad (2)$$

The important condition is if  $\mu$  does not equal  $\Pi$ ,  $w$  will not equal  $w_{mkt}$ . The risk aversion coefficient in the reverse optimization process acts as scaling factor for the reverse optimization estimate of excess return and is calculated as follows

$$\lambda = \frac{E(r) - r_f}{\sigma^2} = \frac{\text{Risk premium}}{\text{Variance}} \quad (3)$$

This scaling factor characterizes the expected risk-return trade-off and is the rate at which more return is required for more risk.

Then model mixes forecasts with equilibrium returns in a Bayesian analytic framework. It translates views into explicit security return forecasts and new covariance matrix suitable for conventional mean-variance portfolio optimizer. Fed with new inputs, optimizer produces portfolios tilted to reflect investor's views.

Using Implied Equilibrium Return Vector and the Black-Litterman Formula 4 the new Combined Return Vector (E[R]) is calculated as follows.

$$E[R] = [(\tau \Sigma)^{-1} + P' \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q] \quad (4)$$

Where: E[R] is the new (posterior) Combined Return Vector (N x 1 column vector);

$\tau$  is a scalar;

$\Sigma$  is the covariance matrix of excess returns (N x N matrix);

$P$  is a matrix that identifies the assets involved in the views (K x N matrix or 1 x N row vector in the special case of 1 view);

$\Omega$  is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view ( $K \times K$  matrix);

$\Pi$  is the Implied Equilibrium Return Vector ( $N \times 1$  column vector); and

$Q$  is the View Vector ( $K \times 1$  column vector).

Since market views are always taken into account there is a little chance to run into unstable or corner solutions. On the other hand if investor has some strong views which can rule the market view, this is because the model gives an opportunity to results to be adjusted to these views.

As it was mentioned before the Black-Litterman model allows the investor views to be expressed. These views can be expressed in two forms: absolute and relative. The absolute view states some equity has some excess return and certain confidence level. Whereas relative view is expressed in the form of some asset which outperforms the other asset by some certain rate and with corresponding confidence. In comparison to absolute views, relative views are more close to investor' view about different assets.

Here we come to the conversion of views to the input which can be applied in Black -Litterman model. The model does not require the investors to specify their views on all assets. However, views which are uncertain give random, unknown, independent, normally-distributed Error term vector ( $\varepsilon$ ) which has a mean of 0 and covariance matrix  $\Omega$ . Therefore, a view will be shown in a form of  $Q + \varepsilon$  on matrix

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \tag{5}$$

Almost in all cases the error term ( $\varepsilon$ ) is equal to positive or negative number other than 0, except for the situation when investor is hundred percent sure about his expressed view. The Error term vector cannot straightly be included into the Black-Litterman formula. But if we take the absolute difference from the error term's expected value of 0, then it can be included into the given formula, in other words this difference called variance of each error term.  $\Omega$  is known as a diagonal covariance matrix with zeros in all of the off-diagonal positions which is derived from variances of the error terms ( $\omega$ ). Because the model thinks that the views are independent from each other, off-diagonal elements of  $\Omega$  are equal to zero. Thus we have relation between two measures, the larger the variance of error term, the greater the uncertainty of the view. Most difficult process is to determine individual variances of the error terms, which in their turn constitute the diagonal elements of  $\Omega$

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix} \tag{6}$$

On the matrix Formula 7 below we can see matrix  $P$  which serves as matching tool of expressed views in column  $Q$  to specific assets. We see that each view results in a  $1 \times N$  row vector.

$$P = \begin{bmatrix} P_{1,1} & \cdots & P_{1,n} \\ \vdots & \ddots & \vdots \\ P_{k,1} & \cdots & P_{k,n} \end{bmatrix} \tag{7}$$

For the purpose of specifying the values of matrix  $P$  model provides two ways. The first one *a market capitalization weighting scheme*; this scheme shows that the relative weighting of each assets is proportional to the asset's market capitalization divided by the total market capitalization of underperforming or outperforming assets of that particular view. The second one is so-called *equal weighting scheme* under which the weighting is proportional to 1 divided by the number of respective assets which underperformed or outperformed. However, this scheme may outcome in tracking error, the reason for that is ignorance of the market capitalization of the assets involved in the view.

The next thing to do is calculation of the variance of each individual view portfolio. Here, the variance of an individual view portfolio is denoted by  $p_k \Sigma p_k'$ ,  $p_k$  is a single  $1 \times N$  row vector from  $P$  Matrix which corresponds to the  $k$ -th view, and  $\Sigma$  is the covariance matrix of excess returns. It should not be forgotten that the respective variance of each individual portfolio is vital source of information concerning the certainty, lack of the level of confidence that should be placed on a view. That is important since we use this information to revisit the variances of the error terms forming the diagonal element of  $\Omega$ .

The most abstract and complicated parts to specify parameters of the model are the scalar ( $\tau$ ) and the uncertainty in the views. To make the Black-Litterman model more standardized we should make an assumption on the value of the scalar. Guangliang He and Robert Litterman adjusted the confidence of a view so that the ratio  $\tau/\omega$  is equal to variance of the view portfolio ( $p_k \Sigma p_k'$ ). If we take the general case the covariance matrix of the error term ( $\Omega$ ) is represented in the following way in Formula 8:

$$\Omega = \begin{bmatrix} p_1 \sum (p_1') * \tau & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & p_k \sum (p_k') * \tau \end{bmatrix} \tag{8}$$

After calculation of the covariance matrix of the error term, the actual value of the scalar is not relevant since only the ratio  $\omega/\tau$  enters the model. Making sure that the scalar value and the covariance matrix of the error term are available, we are at the end point to derive new combined return vector. That we can get if we enter all inputs into the Black-Litterman formula and solve the unconstrained maximization problem.

**BACKTEST**

Prechter in his publications mostly covers the major asset classes which have significant influence on financial market, so portfolios made with his predictions could enjoy high level of diversification. Our backtest cover quarter returns of the following assets:

*Domestic fixed income*

- Government bonds - 30 year US Treasury Bond

Futures.

*Domestic equity*

- Dow Jones Industrial Average Total Return Index;
- Large-caps - S&P 500 Total Return Index;
- Small-caps - Russell 2000 Total Return Index.

*Gold*

- Historical gold spot prices.

*Silver*

- Historical silver spot prices.

*Crude Oil*

- Crude Oil Futures.

*REITS*

- FTSE/NAREIT US Real Estate Index.

*Other Commodities*

- The Standard and Poor's Goldman Sachs Commodity Index (S&P GSCI).

Black-Litterman requires market capitalization measures as well as historical returns. Equities capitalization was calculated directly from weights and capitalizations of respective index members. Market capitalization for 30 year Treasury Bonds was proxied by value of open market interest. Market capitalization of gold was taken from all investable gold of US institutions (data provided by World Gold Council). For silver and futures, we used exchange data.

We compared two types of Black-Litterman portfolios with other methods of allocation, namely passive market portfolio (cap-weighted benchmark returns), equally-weighted portfolio and classical 60/40 stock/bond allocation. The Black-Litterman model provides us with two types of portfolio:

- ✓ The Black-Litterman Equilibrium Returns portfolios without views;
- ✓ The Black-Litterman with Prechter's views specified portfolios.

Following (Mikaelyan, 2012) approach we chose from 5 portfolios efficient frontier for every type of Black-Litterman portfolio which are:

- the minimum risk portfolio (minrisk);

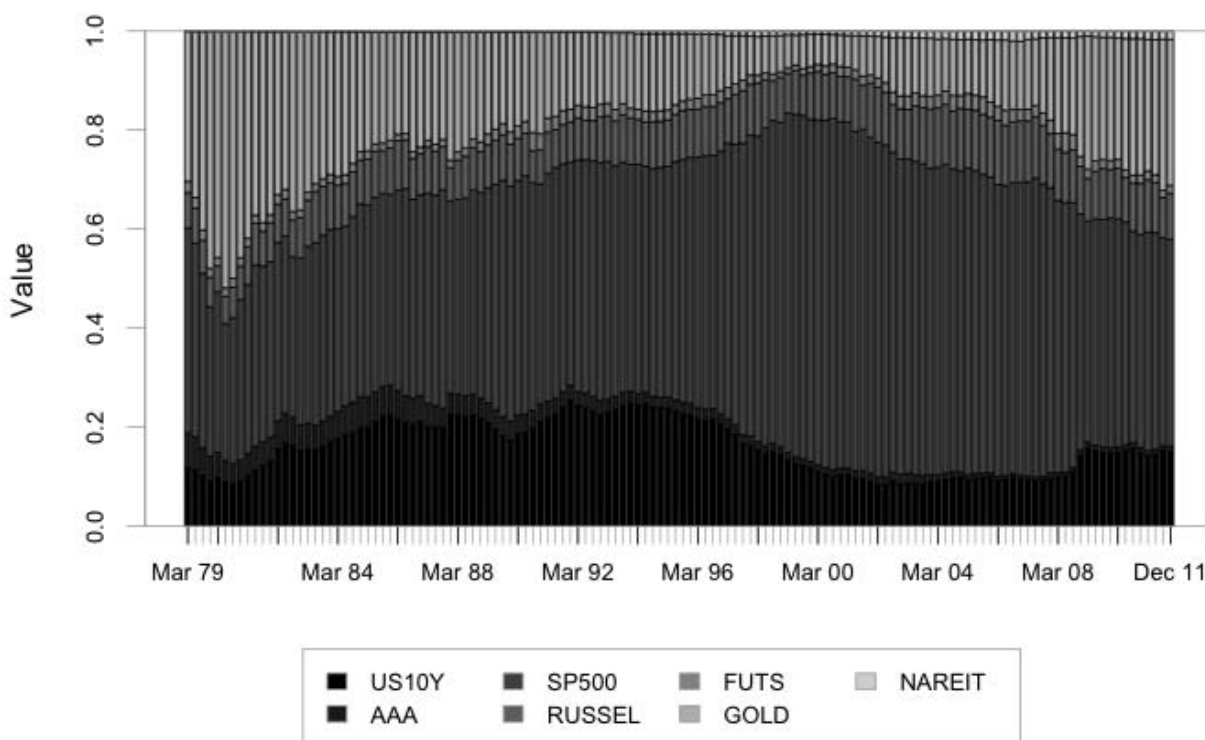


Figure 1. Market portfolio historical allocation.

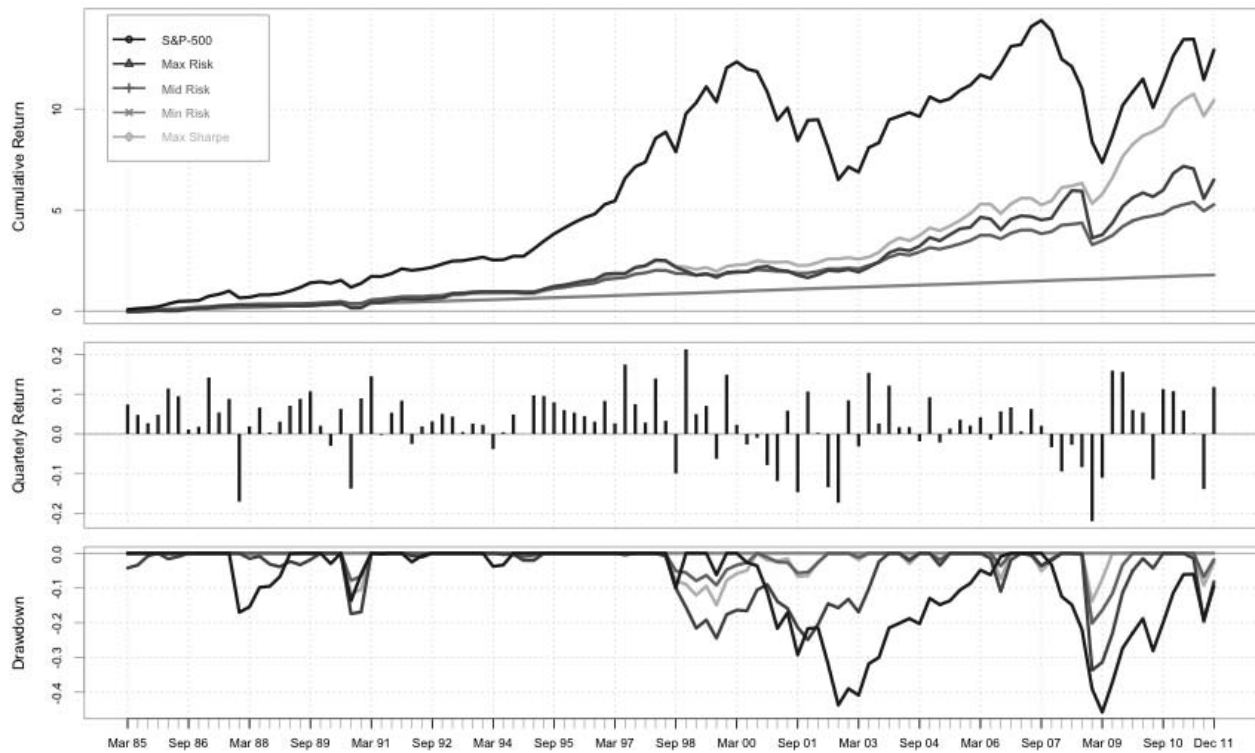


Figure 2. Performance of Black-Litterman active portfolios with views.

- the maximum risk portfolio (maxrisk);
- the medium risk portfolio (midrisk);
- the middle between minimum and medium risk portfolio (minmidrisk);
- the middle between medium and maximum risk portfolio (midmaxrisk).

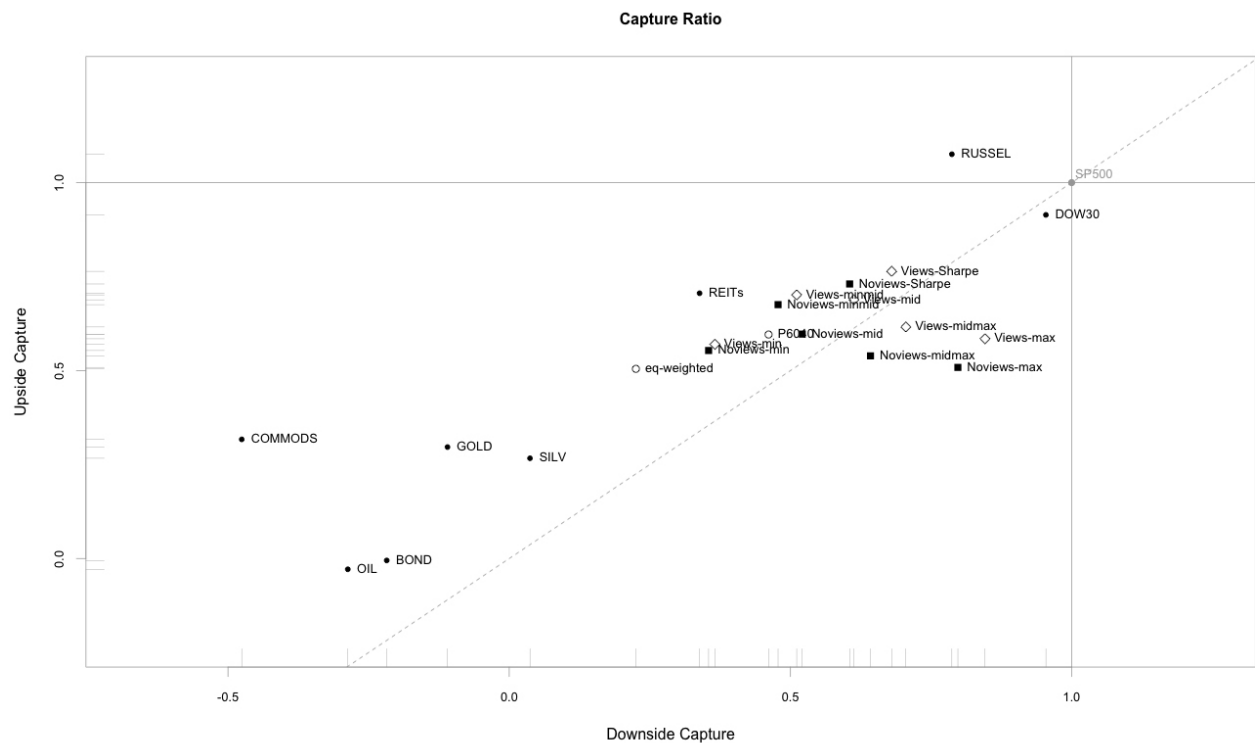
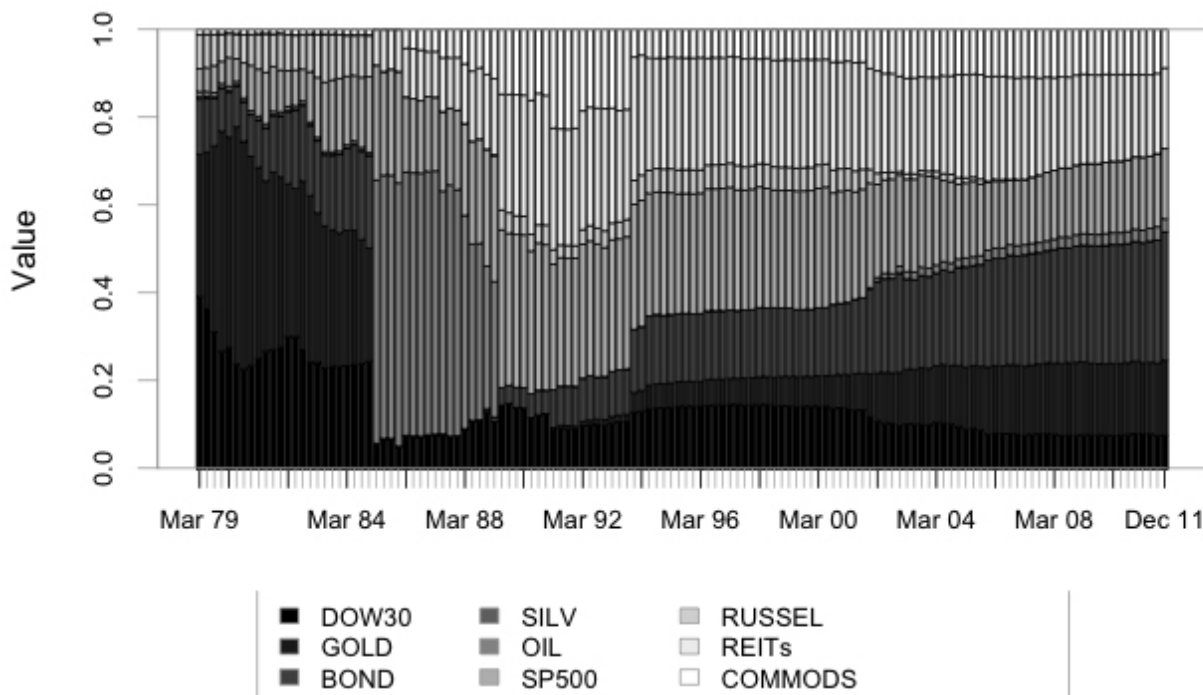


Figure 3. Capture ratios of basic assets in universe (dots), equilibrium Black-Litterman portfolios (squares), Black-Litterman portfolios with Prechter's views (diamonds).



**Table 1. Performance ratios.**

Portfolios		Calmar	Sterling	Drawdown	Sortino
Minimum risk	Prechter	0,1252	0,1057	0,5417	0,3601
	Equilibrium	0,1182	0,1001	0,5544	0,3495
Min-med risk	Prechter	0,3111	0,2307	0,2870	0,6881
	Equilibrium	0,3106	0,2297	0,2839	0,8013
Medium risk	Prechter	0,2705	0,2008	0,2885	0,5805
	Equilibrium	0,2428	0,1796	0,2839	0,6246
Med-max risk	Prechter	0,1449	0,1145	0,3775	0,3601
	Equilibrium	0,1263	0,0993	0,3669	0,3496
Maximum risk	Prechter	0,0740	0,0609	0,4677	0,2235
	Equilibrium	0,0555	0,0457	0,4677	0,1910
Maximum Sharpe ratio					
Prechter		0,2970	0,2214	0,2930	0,5960
Equilibrium		0,1182	0,1001	0,5544	0,3494
Market Portfolio		0,3105	0,2296	0,2839	0,7489
60/40 portfolio		0,2931	0,2095	0,2510	0,6610
Equally-weighted portfolio		0,1838	0,1466	0,3950	0,5551



**Figure 4.** Minimum level of risk Black-Litterman Portfolio with Prechter's views.

Having chosen this set of portfolios, we will define which portfolios type of portfolio allocation method will be better in corresponding level of risk. The same technique we can apply to other portfolios. They will be subdivided by the risk-interval they belong to. Performance of the created portfolios was assessed with Calmar, Sterling,

Sortino ratios, and maximum drawdown measure (Table 1). We also used capture ratios and equity curves visual analysis to determine best portfolios (Figure 2 and 3). In the coming section we define which type of portfolio is better for a given level of risk on the basis of performance analysis ratios discussed above.

Equity curve and ratios analysis show that all Black-Litterman portfolios with views have significantly lower drawdowns and returns than benchmark (Figure 2). Moreover, with-views Black-Litterman portfolios are dominating equilibrium ones on upside/downside capture space (Figure 3), which may suggest that Prechter views are actually adding value. The evidence is supported by Calmar, Sterling,

Sortino ratios and maximum drawdown measures (Table 1).

From the other hand, almost all one-asset concentrated portfolios outperform all Black-Litterman-diversified portfolios in terms of upside/downside capture ratio. The observation probably owes to the fact, that concentrated portfolios are 100% invested in assets, which are well-known di-

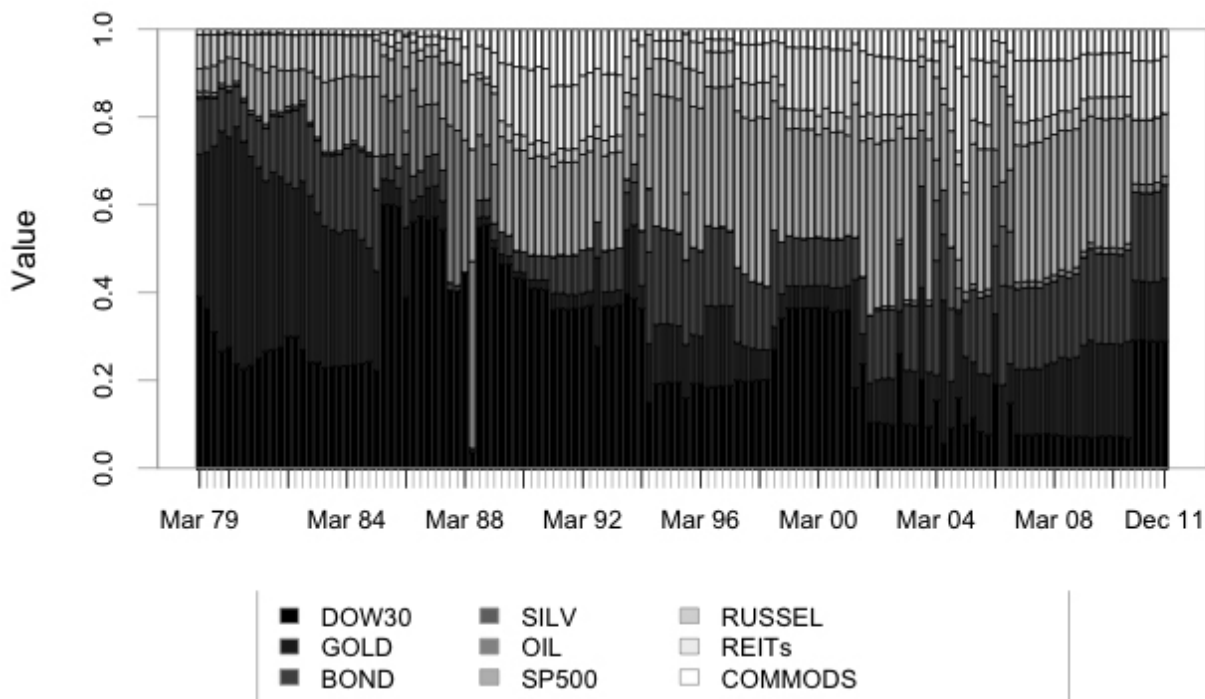


Figure 5. Minmid risk level of Black-Litterman portfolio with Prechter's views.

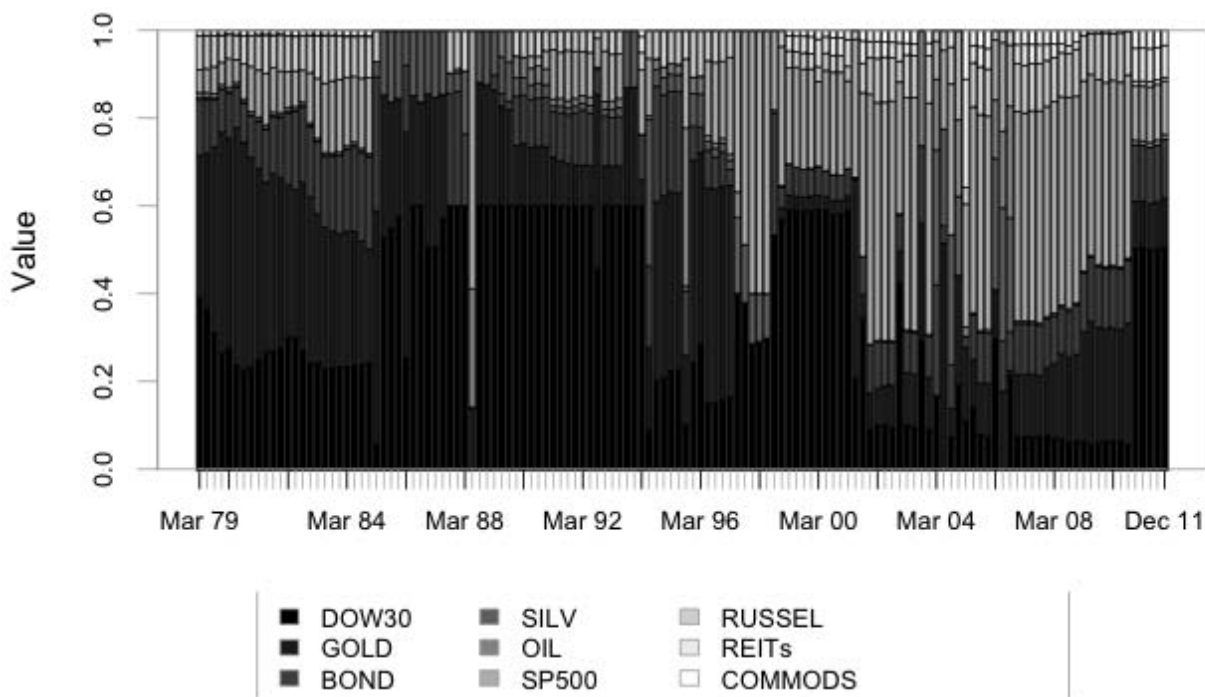


Figure 6. Medium level of risk Black-Litterman portfolio without views.

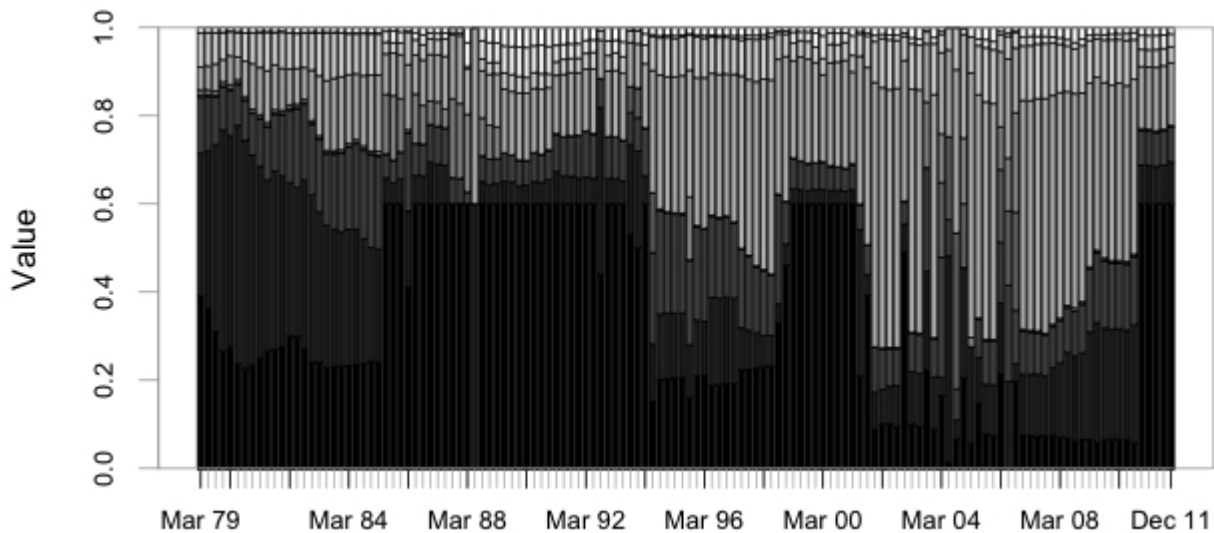


Figure 7. Black-Litterman Sharpe portfolio with Prechter's views.

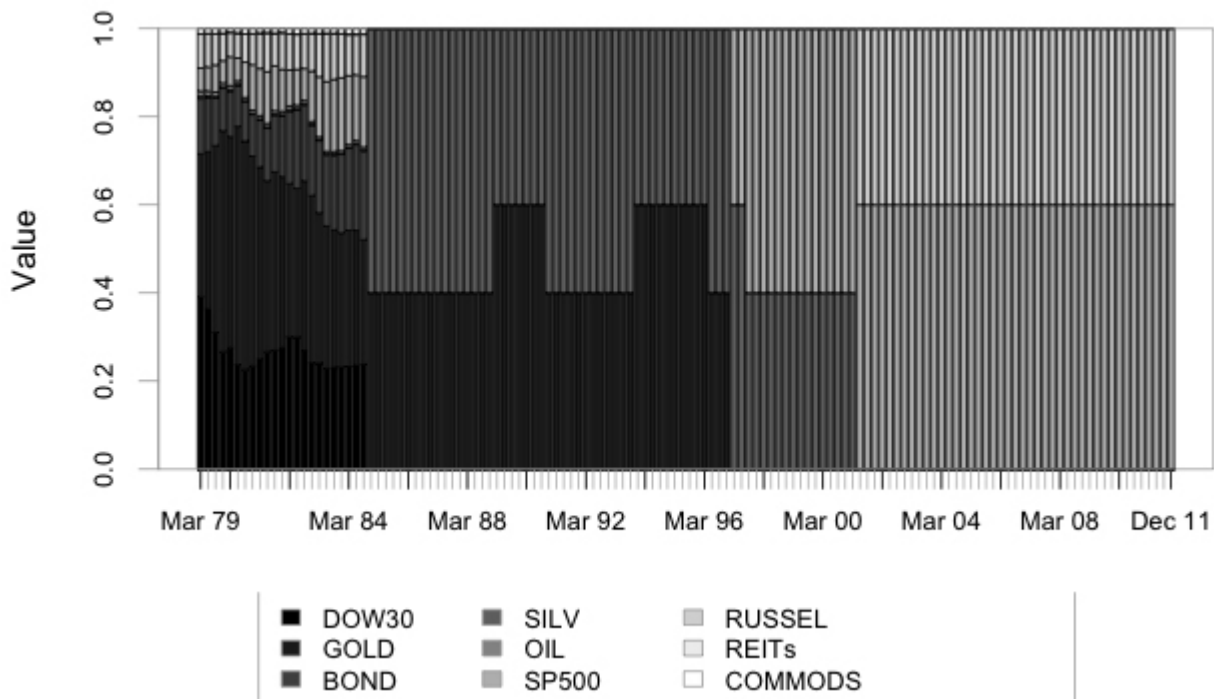


Figure 8. Maximum risk level Black-Litterman portfolio without views.

versifiers to big US caps, selected as benchmark. Nevertheless, mid-max and max Black-Litterman portfolios, both with and without views, appear to be inefficient in terms of upside/downside ratios, as they bear more than one unit of downside risk for a unit of upside risk taken.

Based on ratio analysis, Black-Litterman minimum-to-medium risk portfolio without views shows better results than same risk level portfolio with Prechter's views. While with-views portfolio has drawdown-adjusted return (i.e. the best Calmar and

Sterling ratios) slightly better than all other portfolios we analyze, equilibrium Black-Litterman portfolio pays the most for its downside semideviation (has the best Sortino ratio). Upside/downside analysis shows that two portfolios are very close to each other; with-views portfolio seem to be a bit riskier, as it captures both more upside and downside.

Medium-to-maximum portfolios have varying scores under different measures: while portfolio with views have better risk adjusted return (Calmar, Sterling and Sortino ratios), portfolio without views has

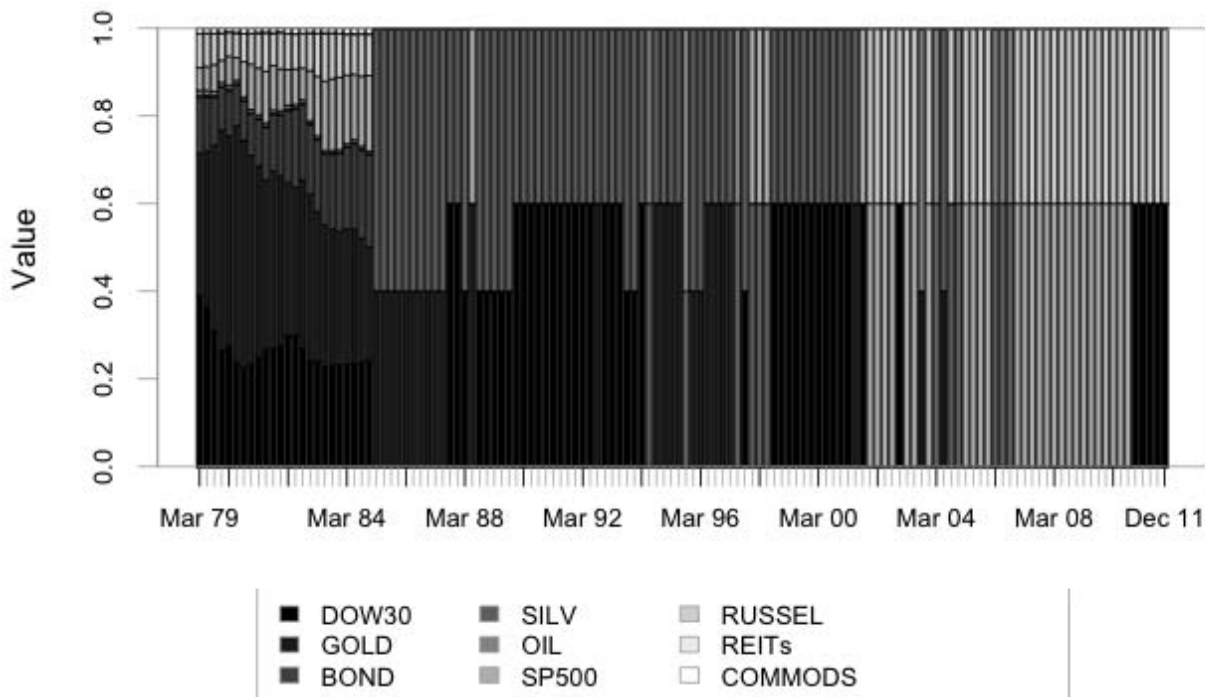


Figure 9. Maximum level of risk Black-Litterman portfolio with Prechter's views.

smaller Max Drawdown and higher upside potential ratio.

All medium-to maximum and maximum-risk portfolios, both equilibrium (Figure 8) and Prechter's (Figure 9) are inefficient in terms of upside/downside capture (Figure 3). Clearly, these portfolios are overconcentrated and exhibit very sharp changes in portfolio allocations, which suggest high transaction costs. Still, even here we see that Prechter's portfolios are better in terms of upside and worse in terms of downside, hence more risky.

Having considered asset allocation methods and their corresponding assets combination, we should pay attention to approximate returns they could bring. Based on historical returns, we have analysed all portfolios with trading simulation approach, where the initial investment was 1,000,000 USD. We have received the result that *Black-Litterman portfolio with Prechter's Views at Minimum Medium risk* have beaten *Market Portfolio*, bringing 16 822 472,57 USD versus 16 250 805,14 USD of Market portfolio.

**CONCLUSION**

Almost at all risk levels with-views portfolios have advantage over equilibrium ones in terms of drawdown-adjusted returns (but not drawdowns). This advantage is smaller at portfolios of lowest and highest risk levels, and peaks at medium-risk and maximum Sharpe ratio portfolios. Equilibrium portfolios, on overall, have smaller drawdowns, and bet-

ter Sortino ratios for all lower-than-medium risk portfolios. More risky with-views portfolios are dominating by all possible measures. This is consistent with upside/downside capture ratios analysis, as portfolios built on Prechter views seem to have riskier returns than equilibrium portfolios of comparable risk levels.

On overall, risky Black-Litterman portfolios (medium-to-maximum and maximum), both equilibrium, and with-views, seem to be worse than market portfolio, capturing less than one unit of market upswings, and more than one unit of market downswings. While one might blame overconcentration of portfolios on the right side of efficiency frontier, the problem is not in overconcentration as it is: clearly, portfolios, 100% invested in any of alternative assets proved to be better in upside/downside to equity, than almost any other portfolio in the investment universe. As Black-Litterman portfolios are bounded by capitalization, overconcentrated riskier-than-average Black-Litterman portfolios are usually 100% invested in equity. Prechter views sometimes tilt this allocation toward alternatives, and this is, probably, one of the reasons why with-views portfolios are better. But this advantage works only when alternatives are mixed in diversified portfolios.

We used Black-Litterman optimization approach to obtain diversified portfolios, resembling portfolios of real market participants. Comparing these portfolios to various benchmarks by various measures we have found that Prechter's forecasts have at least marginal value for market participants.

## REFERENCES

- Allocation Model: Three Years of Practical Experience. Fixed Income Research.
- A. J. Frost and R. R. Prechter. "Elliott Wave Theory", seventh edition 1995.
- Bevan, A., and Winkelmann, K. (1998). Using the Black-Litterman Global Asset.
- Black, Fischer, and Robert Litterman. Global Portfolio optimization. *Financial Analysts Journal*, September/October 1992, pp. 28–43.
- Black, Fischer, Litterman, Robert, "Global Portfolio Optimization, *Journal of Fixed Income*, 1991.
- French, Craig W. (2003). "The Treynor Capital Asset Pricing Model". *Journal of Investment Management* 1 (2): 60–72. SSRN 447580.
- Goldman Sachs Asset Management Working paper, 1999, Goldman, Sachs & Company, December.
- George S. Atsalakis. Elliott Wave Theory and neuro-fuzzy systems, in stock market prediction: The WASP system.
- He, Guangliang, Litterman, Robert, "The Intuition Behind Black-Litterman Model Portfolios".
- He, Guangliang, and Robert Litterman. "The Intuition Behind Black-Litterman Model Portfolios". *Investment Management Research*, Goldman, Sachs & Company, December 1999.
- Idzhorek, Thomas, "A Step-By-Step guide to the Black-Litterman Model, Incorporating User-Specified".
- Jerry A. Miccolis. Next generation Investment Risk management: Putting the 'Modern' back into Modern Portfolio theory".
- Jay Walters, "The Black-Litterman Model in Detail. Working paper, 2008.
- Litterman, Robert, and the Quantitative Resources Group, Goldman Sachs Asset Management. *Modern Investment Management: An Equilibrium Approach*. New Jersey: John Wiley & Sons, 2003.
- Markowitz, H. M. (March 1952). "Portfolio Selection". *The Journal of Finance* 7 (1): 77–91.
- Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons. (reprinted by Yale University Press, 1970, ISBN 978-0-300-01372-6; 2nd ed. Basil.
- Roy, Arthur D. (1952). "Safety First and the Holding of Assets". *Econometrics* 1952 (July): 431–450.
- Meucci, Attilio, "The Black-Litterman Approach: Original Model and Extensions." *The Encyclopedia of Quantitative Finance*, Wiley, 2010. Prechter R.R. "Beautiful Pictures", 2003
- Prechter R.R. "At the crest of the Tidal wave", 1995.
- Poser S. W. *Applying Elliott Wave Theory Profitably*, 2002.
- Quantitative and Traditional Portfolio Construction. " *Journal of Asset Management*, Vol 1, 2, 138–150.
- Ross, Stephen (1976). "The arbitrage theory of capital asset pricing". *Journal of Economic Theory* 13 (3): 341–360. Stephen J. Brown. *William Peter Hamilton's Track Record the Dow Theory: Reconsidered*. 1998.
- Theil, Henri, *Principles of Econometrics*, Wiley and Sons, 1971.